PSYCHOLOGICAL AND DIDACTIC ASSUMPTIONS ON MATHEMATICAL PROBLEM SOLVING

PRESSUPOSTOS PSICOLÓGICOS E DIDÁTICOS PARA A RESOLUÇÃO DE PROBLEMAS MATEMÁTICOS

PRESUPUESTOS PSICOLÓGICOS Y DIDÁCTICOS PARA LA RESOLUCIÓN DE PROBLEMAS MATEMÁTICOS

Fatima Aparecida Souza FRANCIOLI[1]
Nilza Marcia Mulatti SILVA[2]

ABSTRACT: This article examines analyses from a master's research project in the field of education, aiming to present psychological and didact assumptions regarding mathematical problem solving. The research project, which is theoretical-methodological, discusses the difficulties of students, in the early years of elementary school, in solving mathematical problems. To that end, the research of Vigotski and Kalmykova have been used for psychological and Saviani didactic matters, respectively. The results appoint that problem solving requires students to transcend from descriptive to explanatory procedures and thus become aware of their actions. Moreover, to better value problem resolution, teachers must propose that students explicit their process besides just providing the correct answers, favoring the mobilization of his ideas and arriving at thinking through concepts.

KEYWORDS: Vigotski and Kalmykova. Saviani. Elementary School. Subtraction.

RESUMO*: Este artigo tomou como recorte as análises proferidas em uma pesquisa de mestrado na área de ensino e tem como objetivo apresentar os pressupostos psicológicos e didáticos referentes à resolução de problemas matemáticos. A pesquisa, de cunho teórico-metodológico, discute as dificuldades dos alunos, dos anos iniciais do ensino fundamental, ao resolverem problemas matemáticos. Para tanto, apoiou-se nos estudos de Vigotski e Kalmykova para as questões psicológicas e estudos de Saviani para as questões didáticas. Os resultados apontaram que a solução de problemas exige que o aluno transcenda dos procedimentos descritivos para os explicativos e, assim, tome consciência de suas ações. Além de que o professor, ao valorizar o processo de resolução, ademais da resposta correta do problema, deve propor ao aluno a explicitação do procedimento realizado, favorecendo a mobilização de suas ideias e chegando ao pensamento por conceitos.*

PALAVRAS-CHAVE*: Vigotski e Kalmykova. Saviani. Ensino Fundamental. Subtração.*

[1] Paraná State University (UNESPAR), Paranavaí – PR – Brazil. Professor at the Postgraduate Program Stricto Sensu, Academic Master's level in Interdisciplinary Teacher Formation – PIPIFOR. Doctorate in School Education (UNESP). ORCID: https://orcid.org/0000-0001-8373-7056. E-mail: fas.francioli@hotmail.com
[2] Paraná State University (UNESPAR), Paranavaí – PR – Brazil. Professor and Pedagogical Coordinator of the Municipal Education Network of Alto Paraná. Master's in Interdisciplinary Teacher Formation (UNESPAR). ORCID: https://orcid.org/0000-0003-3895-3029. E-mail: nmulatti29@hotmail.com

RESUMEN: *Este artículo tomó como punto de referencia, los análisis manifestados en una investigación de maestría en el área de enseñanza y tiene como objetivo presentar los presupuestos psicológicos y didácticos referentes a la solución de problemas matemáticos. La investigación se basó en el modelo teórico-metodológico, discute las dificultades de los estudiantes, en los primeros años de la escuela primaria, para resolver problemas matemáticos. Para esto, se utilizaron los estudios de Kalmykova para la parte psicológica y los estudios de Saviani para las cuestiones didácticas. Los resultados apuntaron que la solución de problemas exige que el alumno trascienda de los procedimientos descriptivos a los explicativos y así tomar conciencia de sus acciones. Además, que el profesor, al valorar el proceso de resolución y la respuesta correcta del problema, debe proponer al alumno la explicación del procedimiento realizado, favoreciendo así, la movilización de sus ideas y llegando a un nivel de síntesis de análisis de conceptos.*

PALABRAS CLAVE: *Vigotski e Kalmykova. Saviani. Enseñanza Fundamental. Sustracción.*

Introduction

In the work of pedagogical practice, the teaching and learning relationship is part of an ongoing process between teacher and student. In order to approach this process, the present article took as an outline the analyzes given in a master's research in the teaching area in which the difficulties of students, from the early years of elementary school, when solving mathematical problems are discussed.

Many say that learning mathematics is not easy, however, the question is to seek answers to demonstrate how to solve some questions: Why do students who know how to solve algorithms often do not know how to apply them to solve mathematical problems? Why are some students able to interpret and others not? In search of these answers, this study aims to present the psychological and didactic assumptions regarding the resolution of mathematical problems, with an emphasis on subtraction. With a theoretical-methodological nature, the study is based on Vigotski and Kalmykova for psychological issues and on Saviani for didactic issues.

Zinaida Ilinichna Kalmykova[3] (1977), based on cultural-historical psychology, continued the studies developed by Vigotski, specifically, in the area of mathematics with an emphasis on learning and development. Dermeval Saviani (1997), Brazilian author renowned for his studies in the area of theories and history of education, defined five categories of knowledge that he considers necessary for the student's development: *mastery of curriculum content, didactic-curricular knowledge, pedagogical knowledge, social-historical conditions,*

[3] Collaborator at the Institute of Psychology of the USSR Academy of Pedagogical Sciences (LURIA *et al.*, 1977, p. 9).

attitudinal knowledge. For Saviani, these categories establish the dimension of knowledge that the teacher needs to master to develop good teaching.

The relationship between these three authors is in the materialist philosophical structure, in which they study school education as a promoter of human development.

Psychological Assumptions for Learning and Solving Mathematical Problems

One of the initial answers that guided the path taken by the research was based on the ideas of Dante (2008), who says that the student will only be able to solve mathematical problems if he masters the concepts of addition, subtraction, multiplication and division. Thus, Vygotski and Kalmykova sought to understand how the appropriation of concepts occurs.

From the perspective of cultural-historical psychology, children's learning begins long before they attend school, however, it is through formal education that students come into contact with the concepts organized in different areas of knowledge that make up the curriculum. From this point onwards, we will try to define the formation of concepts.

Vigotski (2001), in his studies, highlights the existing relationships between spontaneous concepts and scientific concepts. For him, *spontaneous concepts are those* formed by direct communication with people with whom the child lives, in a free way, without defined intentionality. Differently, *scientific concepts* are developed through intentional and systematized mediation, which is the exclusive responsibility of school education. Vigotski (2001, p. 218, our translation) states that "spontaneous concepts enable the appearance of non-spontaneous concepts through teaching" and that "the formation of scientific concepts in the same measure as the spontaneous ones, does not end, but only begins when the child assimilates for the first time a new meaning or term for him, which is a vehicle for a scientific concept” (Idem, p. 265, our translation). For the author, it is not because the content is addressed at school that it reaches the scientific conceptual level, even knowing that it is the role of school education to provide activities capable of transforming *spontaneous concepts* into *scientific* ones, that is, transforming the spontaneous thinking of students in intellectual thinking.

And how does the child appropriate the concepts? Before answering this question, it is necessary to clarify that Vigotski (2001) divided the path of thought into three main basic stages: *syncretic thinking*, *thinking by complex* and *thinking by concept*. At the stage of syncretic thinking, the main characteristic is a tangle of ideas without internal foundation but linked to the child's impression of things; “[...] it is the formation of an uninformed and unordered plurality, the discrimination of a heap of various objects at the moment that this child is faced with a problem” (VIGOTSKI, 2001, p. 175, our translation). For the author, *thinking by complex* “leads to the formation of bonds, the establishment of relationships between different concrete impressions, the unification and generalization of particular objects, the ordering and systematization of the child's entire experience” (Idem, p. 178, our translation). Thus, at this stage, the child begins to make the first relationships and presents as a basis the link with the concrete between the elements. At this stage, the first generalizations begin. We cite as an example the act of putting pieces together according to the attribute color or the attribute shape.

The last and desired stage to be reached is *thinking by concept*.

> [...] the concept in its natural and developed form, presupposes not only the combination and generalization of certain concrete elements of experience, but also the discrimination, abstraction and isolation of certain elements, and also the ability to examine these elements discriminated and abstracted out of the concrete and factual bond in which they are given in the experience (VIGOTSKI, 2001, p. 220, our translation).

In addition to this concept, Vigotski (2001, p. 226, our translation) states that “the concept arises when a series of abstracted attributes is synthesized again, and when the abstract synthesis thus obtained becomes fundamental to thought”. Through this synthesis, the child perceives and becomes aware of the reality that surrounds them. However, “the *concepts* do not emerge mechanically as a collective photograph of concrete objects” (VIGOTSKI, 2001, p. 237, our translation). Its formation always arises in the process of solving a problem that arises in thought. The *concept* will emerge from the solution of this problem, therefore, from this statement we confirm the relevance of the act of problematizing school contents.

For the development of *scientific concepts* to occur, tasks are needed that enable the student's thinking to turn more to mental activity than to the sensory object. In this case, the acquisition of *scientific concepts* follows the opposite path from the spontaneous ones, developing through a deductive process of complex and superior properties to elementary and inferior properties. In other words, the tasks have as their starting point the mental activity, based on the abstraction of knowledge that promote the appropriation of the concept. Upon reaching this level of conceptual thinking, it becomes possible to relate this concept to spontaneous knowledge, present in lived experiences.

The domain of the thought act reveals the student's level of psychic development, that is, he manages to convert his psychic functions, such as perception, memory, voluntary attention and thought itself, into objects of consciousness. It means, so to speak, that this student is in intense mental activity, fully aware of the thought process to the point of mastering it.

In this direction, continuing Vygotsky's research, Kalmykova (1977), as a Soviet researcher, developed in the mid-twentieth century, together with Leontiev, Luria, Zankov and other collaborators of historical-cultural psychology, different studies to contribute to the work of teachers and improve the learning of children in the early years of elementary school. For this researcher, it was essential to investigate teaching methods used by good teachers, comparing them and observing their effectiveness in solving mathematical problems.

According to Kalmykova (1977), problem solving requires much more than knowing numbers and operating techniques, it requires knowledge of several concrete and abstract concepts that reflect the quantitative relationships between objects. Therefore, to solve a problem well, it is necessary to have syntheses at the level of complex analyses. Even in a simple problem, data can be interconnected in different ways, which requires elaborate reasoning to solve. In a compound problem, which needs to be solved in more than one step, choosing the operation to be used becomes more difficult, as the student needs to choose the right numbers and define their possible combinations. “This preliminary analysis is essential for a correct solution of complex problems” (KALMYKOVA, 1977, p. 10, our translation).

Another important point, highlighted by Kalmykova (1977), refers to the statement that, in the formation of concepts, the more diversified the concrete material, the easier the abstraction process will be. However, she recognizes the impossibility of carrying out a sensory experience with all materials, which is why those that enhance the expansion of the studied concept should be prioritized.

Kalmykova (1977) analyzed the practice of one of the best teachers in a Moscow school, D.V. Petrova, class I teacher. From the reports presented, there are signs that this is the first year of elementary school. Among her observations, the author highlights that, even before the children started to read and learn the first mathematical contents, the teacher made available to them a variety of non-school materials and objects. These concrete materials, according to the author, facilitated the transition to abstraction to the concept of number, mathematical operations and problems.

Another relevant approach observed by the author and carried out by teacher Petrova refers to the use of drawings as a means of consolidating the content. For example, the number 5 was related by the teacher to five objects, the teacher guided the child, so that he/she did not form a single specific connection, that is, to relate the word 5 only with that amount of concrete objects. We can infer, with Kalmykova (1977, p. 16), that the targeting should be based on the gradual decrease in the number of objects and signs, starting to use them only to introduce new concepts, or, when necessary, "constitute and consolidate connections". The author also advises that, in order to guide children towards generalization, we can make use of images, as they are based on concrete reality, but they are not this reality.

Kalmykova (1977) advises that the efficient work on the formation of concepts is not limited to the first studies but extends through all the years of schooling. In this sense, we consider correct the fact that the concept of subtraction starts in early childhood education and extends to elementary school, as appropriation does not occur in a punctual and complete way at once.

Another direction that we consider important in relation to the analysis of errors refers to the necessary attention to the students' way of thinking and the essential concepts to understand certain school content. We highlight the mediations to make the student recognize the error, think about "why" he made a mistake, change his answer and recognize the correctness. Therefore, it is not enough to show the error and correct the students' answers. Just considering error as part of the process does not advance learning. The advances come from the analysis carried out by the student, through the mediation of the teacher, who perceives that the resolution performed does not match the logic of the proposed activity.

In the more advanced classes, called class II and III, which can be compared to the second and third years of elementary school, the teacher introduced these concepts, asking students to translate the text of the mathematical problem into more abstract terms. They were asked to correctly express the data and the value sought, which required scientific language. In class IV, Kalmykova reports that the teacher began to get the children used to expressing themselves in appropriate mathematical terms not only in the content of the problem but also in its solution. Little by little, she guided students to leave the visual image and move to abstraction, so that they could assimilate the "more complex mathematical categories" (KAMYKOVA, 1977, p. 20).

Having made these considerations, the author clarifies that at first not all students assimilate, but through the systematic work of the teacher on these concepts, all become capable of learning. Therefore, the systematic work on the assumptions of cultural-historical psychology enables not only good students to learn, but everyone involved in the process. Because the reflections and propositions, expressed in theory, are presented as possibilities for the realization of didactic procedures and resources rich in meaning and must appear as essential characteristics in the teaching process.

We return to the initial questions that guide this text: Why do students who know how to solve algorithms often do not know how to apply them to solve mathematical problems? Why are some students able to interpret and others not? In search of these answers, it is relevant to consider that

> [...] the work of forming the concepts necessary for solving problems is a means to increase the effectiveness of the analytical-synthetic activity. But the assimilation of concepts and corresponding mathematical laws does not imply a special ability to solve more complex problems. It is not enough to have notions; it is necessary to be able to use them at the right moment, choosing the necessary notions to solve certain problems. It often happens that a student cannot solve a problem because he does not know how to mobilize the notions he has. Choosing the necessary notions requires a special concentration on the text of the problem, that is, analyzing it (KALMYKOVA, 1977, p. 20-21, our translation).

In this sense, we consider that, in order to be able to interpret and solve a mathematical problem, in addition to learning the concepts of operations, mathematical terms and mastering the resolution of operations, it is necessary to know how to mobilize this knowledge and use it properly.

Kalmykova (1977) warns that the rush to consolidate the habit of solving problems and the lack of time to explain the problem-solving process in detail cause students to have a certain slowness of reasoning. Because they cannot remember the reasoning that leads to the solution, they cannot translate the method used to solve one type of problem onto another problem either. Therefore, it is necessary to emphasize the method used to solve the problems, allowing considerable time for analysis. The author confirms that:

> [...] a conscious assimilation of problem-solving methods not only requires assimilating the corresponding system of arithmetic operations, but also assimilating the form of reasoning through which students analyze the content of a problem and choose certain operations (KALMYKOVA, 1977, p. 24, our translation).

This statement is in line with our investigation, regarding the importance of dedicating special attention to the teaching methods of problem analysis and reasoning during this analysis, that is, to understand the phases that thinking goes through to thinking by concepts. In this sense, we can say that the teacher needs to receive training that covers not only specific content in the area, but also content related to methodologies.

Didactic assumptions and the analysis of students' language manifestations in solving mathematical problems

As discussed above, in the teacher-student relationship lies the founding aspect of school education as a mediator between teaching and learning. This presupposes an educational work that, according to Saviani (1977), must be intentional and produce in each student the knowledge historically developed by humanity.

In this direction, Saviani (1997), when listing the knowledge needed to produce knowledge in the student, defines five categories of knowledge relevant to the teacher's work.

The first category defined by Saviani (1997) seems obvious, as it refers to the "*mastery of curriculum content*", but it is a category that needs to be consolidated. It is noteworthy that no matter the level of performance, the teacher must know extensively the content to be taught, so they need to master the concepts. Knowing the content is the first step, by the way very important, but not enough to transmit knowledge to the student. The second category defined by Saviani (1997) refers to "*didactic-curricular knowledge*"; emphasizes that you need to know how to organize content. Saviani (1997, p. 131, our translation) defines that knowledge needs to be "dosed, sequenced and worked on in the teacher-student relationship". The author states that these first two categories are considered the basic modalities for the teacher to teach efficiently. Saviani (1997) makes this distinction to emphasize the need for teachers to appropriate the pedagogical knowledge produced by the science of education, to know the pedagogical theories that underlie educational policies and that significantly influence teaching practice. The third category refers to "*pedagogical knowledge*", that is, the knowledge produced by the science of education.

It is not possible to get to know the school by studying only the school, as education is inserted in a context in which it is directly influenced by the socioeconomic and cultural situation. Thus, the fourth category deals with the understanding of the "*socio-historical conditions*" that determine the educational task, essential knowledge for thinking about critical education, as criticality permeates the knowledge of the totality.

The fifth category includes "attitudinal knowledge", responsible for establishing coherence between knowing and doing. As the author says, it is not a question of confusing profession with mission, but of adopting an ethical posture. It refers to the attitudes and postures of the role assigned to the teacher, defined by Saviani (1997, p. 136, our translation) as "discipline, punctuality, coherence, clarity, justice and equity, dialogue, respect for the person of the students, attention to their difficulties etc." According to the author, this competence is related to the teacher's identity and personality, but they are objects of formation.

Saviani (1997) defines, through the categories previously presented, the dimension of knowledge that the teacher needs to master. Our position is that the absence of knowledge in any of these categories affects the effectiveness of teaching and compromises the student's ability to apprehend historical knowledge socially produced by humanity.

It is therefore necessary that, based on these didactic assumptions, the teacher adopts teaching and learning situations in their practice whose richness allows the appropriation of scientific knowledge. In this study, we refer to the solving, in the classroom, of mathematical problems.

Given the difficulties presented in mathematical interpretation in solving mathematical problems, checked in classrooms, and the hypothesis that students perform the operations, solve the algorithms, but are not aware of the action they perform, that is, they did not appropriate the concepts scientific research, a pedagogical work[4] was developed, related to the learning of the concept of subtraction, which involved children[5] from nine to eleven years old, enrolled in the fourth year of elementary school in a municipal school located in northwestern Paraná. The purpose was to know the level of awareness of the action of subtracting, through the analysis of the manifestation of oral languages, drawings and object manipulation.

The pedagogical work reported below is part of a sequence of activities carried out, among them, the resumption of contents related to subtraction, interventions that preceded the task requested from students involving solving similar problems, among other actions, are included. We will report below three moments of the classes, considered important to analyze the possible conscious action when solving mathematical problems: oral justification for the choice of mathematical operation, representation through drawing and illustration using manipulative material.

At first, a problem was given to each student and they were asked to answer which mathematical operation they could use to solve it and, mainly, to justify the reason for the choice, that is, the main focus was not on the right answer to the problem, but in the explanation of the thought involved in the resolution.

[4] To access the research. Available: http://ppifor.unespar.edu.br/files/NILZA_MARCIA_MULATTI_SILVA.pdf. Access: 10 June 2021.

[5] According to the ECA – Statute of Children and Adolescents (BRASIL, 2002), a citizen who is under 12 years of age is considered a child.

The oral justifications obtained were of various levels, as can be seen through the following reports: “It solves through addition because it has to count”; “Division because it will put in the bag”; “It's addition because it will join the pages she read with the ones she didn't”; “Subtraction because I withdraw”; “Subtraction because 'lack' to complete the album”. There were also students who were unable to explain their choice, others confused the names of the operations, did not recognize the term "difference" as a result of the subtraction, in addition to naming the addition operation as "too much" and the subtraction operation as " it contained too little”. This lack of use of the correct nomenclature leads us to the fact that the school sometimes reinforces spontaneous knowledge related to the nomenclature of algorithms. We observed, in students who performed better in the tasks, the precise use of the operations nomenclature.

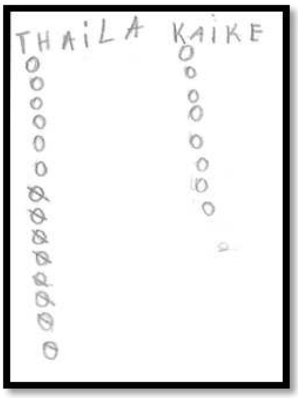
Based on the difficulty in explaining the procedure used to solve problems, and Kalmykova's (1991) defense that drawing would be an intermediary point between the concrete and the abstract, it being necessary to resort to visual material as a basis for the formation of concepts in order not to stop only at the purely formal assimilation of the notions, it was decided to add, in the second moment, drawing as another form of expression, in addition to the algorithm, as a means of consolidating the content.

In this second moment, in order to improve the analysis of the appropriation of the concepts of the operations, through conscious action, six problems from the previous moment were resumed, solved using subtraction. The task consisted of carrying out the operation and drawing it identifying its terms, that is, the minuendum, the subtraendum and the remainder or the difference. When drawing, the student needed to think about what each number used in the algorithm represented and relate it to the proposed problem.

Among the problems, the representation of the student Amanda[6] was chosen, which involves the idea of comparing the subtraction, for the following problem: “Kaike is eight years old and his sister, Thaila, is 14 years old. How many years is Thaila older than Kaike?”

[6] Student names are fictitious.

Figure 1 – Representation of the problem solution



Source: Author (2015)

Observing the above drawing and based on the oral explanation, the student represented the minuendum and subtracting through irregular circles, and the act of subtracting, tracing a line on them. It is noticed that there is no explicit relationship between the minuendum and the subtraendum. When asked where the difference in age between Kaike and Thaila is represented, she explains: "Here there are eight and here there are fourteen". The student knows the meaning of the word but does not identify in her drawing the term "difference" as a result of the subtraction operation.

Drawing as a form of language substantially favored the expression of thought, confirming Kalmykova's (1991) defense that drawing is not the real problem, but expresses the reality thought by the student, and as it is the external manifestation of thought, it can come to be the starting point for abstraction.

Even with these positive points, during the development of the tasks, it was verified that the students had doubts related to how to design the act of withdrawing. Pertinent doubts, because, if I withdraw, how can they remain? Drawing was one more resource, but the action of withdrawing was still compromised. This situation leads us to Kalmykova's statement (1991, p. 12, our translation) that "the necessary psychological basis for the correct formation of concepts is an assimilation that allows for the creation of conditions between the abstract and concrete components of thought, between the word and the image".

In the third moment, the group was composed of six students and manipulated materials were used, for two reasons: due to the students' difficulty in drawing the "removal action", and Kalmykova's (1991) statement that, in the formation of concepts, the more diversified the concrete material, the easier the abstraction process will be.

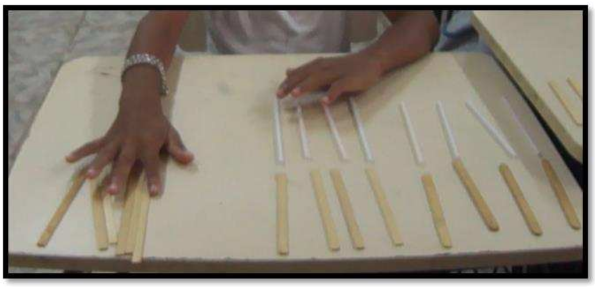
For this purpose, two problems were chosen for the students to represent the subtraction operation, among them, the moment problem previously reported. The task consisted of representing the idea of comparing subtraction. Thaila's age was represented by straws, and Kaike's age by popsicle sticks.

Before solving the task, it was explained that, to solve the problem, they could use subtraction: fourteen minus eight equals two legs (14-8=6). They were asked to perform the operation, using chopsticks and straws, and answer the question: What does the number 8, the subtraendum, represent?

The expected answer, and which supports the idea of comparing the concept of subtraction, is that, when comparing, we remove the quantity that the minuendum and the subtraendum have in common. When comparing amounts, the subtraendum represents the common amount between subtraendum and minuendum, in this case, the number 8 represents the common age between Thaila and Kaike.

All students correctly represented the requested task, however, 1/3 of the students were able to report the procedure performed but could not justify it. Next, two solved tasks will be presented, in which there are signs of conscious action.

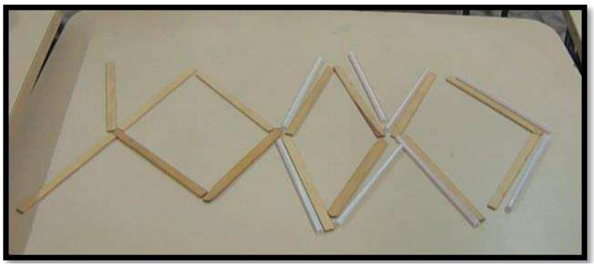
Figure 2 – Problem representation by the student Carlos



Source: Author (2015)

Carlos' explanation: “I put the 14 sticks, then I put the eight straws underneath, then I took these eight. Those straws are Kaike's age. These sticks are from Thaila. We take what is equal and the difference remains”.

Figure 3 – Problem representation by student Breno



Source: Author (2015)

Breno's explanation: "I made Thaila's age with the sticks and then I made Kaike's age with the straws and they resulted in these two triangles (actually irregular quadrilaterals) and there were those left, which made six. So far it's what they have equal and the rest is the difference."

At the end of the task, it was verified that the students Carlos and Breno solved it correctly, were able to explain "what" and "why" they did the representation of the subtraction operation, using straws and sticks. Fabieli did not perform the representation correctly but managed to explain "what" she did. It can be considered an advance. Amanda did it correctly, managed to explain "what" she did, but not "why" she did it. Everton and Daniele did it correctly, but everything indicates that they imitated the accomplishment of the task done by Carlos and could not even explain "what" they did. From this analysis, it can be inferred that Carlos and Breno are aware of the action of subtracting, Fabieli and Amanda are in the process, and Fabieli did not solve it correctly, but managed to explain, for this reason we consider that she is developing the concept. By verbalizing the procedure, she will analyze the resolution and can become aware of her error. Amanda solved the task but could not explain. At this point in the analysis of the task, it was verified that there are signs that the students are in the process of learning the concept of subtraction, but cannot reach the abstraction and generalization, so necessary for the formation of the *scientific concept*. For, in Vigotski (2001, p. 226, our translation), "the *concept* arises when a series of abstracted attributes is synthesized again, and when the abstract synthesis thus obtained becomes fundamental to thought".

Final considerations

The survey data demonstrated the students' difficulty in explaining the procedure used to solve the proposed tasks. However, for Kalmykova (1991), problem solving requires much more than knowing the numbers and operating techniques, that is, it is necessary for the student to appropriate concrete and abstract concepts, reaching syntheses at the level of complex analyses.

In the early years, in addition to oral and written or numerical language, drawing can be used as a form of expression, in addition to the algorithm, as the author's defense confirms when she says that images represent the concrete, but they are not the concrete. This means that the use of drawing, as a didactic procedure, is an intermediate point between the concrete and the abstract, as well as the starting point for abstraction. The drawing represents the external manifestation of thought that, when transposed to the child's thought, was interpreted abstractly by the child. The child, when empirically capturing the analyzed object, reproduces in thoughts the dynamics and structure of that object.

Regarding didactic procedures, the pedagogical categories defended by Saviani (1997) when he emphasizes the mastery of knowledge by the teacher, they need to be listed with the good materials to be used, the forms of language, the tasks performed by the students and the context in that students are included, considering that cultural and economic deprivations affect learning.

We consider that problem solving, in different moments of the class, can implicitly present correct answers, but it is not the understanding of the idea of the operation used to solve the problem. The teacher, when valuing the process of solving, in addition to the correct answer to the problem, must propose to the student the explanation of the procedure performed, favoring the mobilization of their ideas.

REFERENCES

DANTE, L. R. **Aprendendo sempre**: matemática. São Paulo: Ática, 2008.

KALMYKOVA, Z. I. Pressupostos psicológicos para uma melhor aprendizagem da resolução de problemas aritméticos. *In*: LURIA, A. *et al*. (Org.). **Pedagogia e psicologia II**. Lisboa: Estampa, 1977. p. 9-26.

SAVIANI, D. A função docente e a produção do conhecimento. **Educação e Filosofia**, Uberlândia, v. 11, n. 21, p. 127-140, 1997.

VIGOTSKI, L. S. **A construção do pensamento e linguagem**. 1. ed. São Paulo: Martins Fontes, 2001.

How to reference this article

FRANCIOLI, F. A. S.; SILVA, N. M. M. Psychological and didactic assumptions on mathematical problem solving. **Revista Ibero-Americana de Estudos em Educação**, Araraquara, v. 16, n. 4, p. 2637-2651, Oct./Dec. 2021. e-ISSN: 1982-5587. DOI: https://doi.org/10.21723/riaee.v16i4.13612

Submitted: 11/07/2021
Required revisions: 09/08/2021
Approved: 10/09/2021
Published: 21/10/2021