ORGANIZATION OF MATHEMATICS TEACHING FROM THE PERSPECTIVE OF THE ELKONIN-DAVÝDOV SYSTEM[1]

ORGANIZAÇÃO DO ENSINO DE MATEMÁTICA NA PERSPECTIVA DO SISTEMA ELKONIN-DAVÝDOV

ORGANIZACIÓN DE LA ENSEÑANZA DESDE LA PERSPECTIVA DEL SISTEMA ELKONIN-DAVÝDOV

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ABSTRACT: Results of external evaluations, such as PISA, SAEB and SAEP, reveal the low level of proficiency of Brazilian students in Mathematics in basic education. Based on the assumption that learning is the result of educational interactions in which students participate, we look at the ways as it is expected the teaching of mathematical concepts. In order to have more objective data about what is learnt and what is taught, it was necessary to opt for the analysis of a particular situation, in this case, the external evaluations data from a municipality in the state of Paraná, as well as the curricular organization of this subject in the respective municipality. Based on the studies of the organization way of the education from the point of view of Elkonin-Davýdov System, the organization of Mathematics education has to be guided by and for the greatness concept, around which algebra, geometry and arithmetic articulate. This articulation involves an integrated curricular organization, opposed to the fragmentation identified in the organization of education in the municipality in question, which can justify the unsatisfactory results in learning and, simultaneously indicate potential pedagogical directions to overcome this nationwide scenario.

KEYWORDS: Historical-Cultural Theory. Elkonin-Davýdov System. Organization of education. Mathematics learning. Greatness concept.

***RESUMO**: Resultados de avaliações externas, como o PISA, o SAEB e o SAEP, revelam o baixo índice de proficiência dos estudantes brasileiros em Matemática na educação básica. Partindo do pressuposto de que a aprendizagem é resultado das interações educativas das quais os estudantes participam, voltamos nosso olhar para o modo como é previsto o ensino de conceitos matemáticos. Para termos dados mais objetivos sobre o que se aprende e o que se ensina, foi necessário eleger uma situação particular para análise, neste caso, os dados das avaliações externas de um município paranaense, bem como a organização curricular dessa disciplina no respectivo município. Com base nos estudos sobre o modo de organização do*

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ensino sob a perspectiva do Sistema Elkonin-Davýdov, a organização do ensino de Matemática deve orientar-se pelo e para o conceito de grandeza, em torno do qual articulam-se álgebra, geometria e aritmética. Essa articulação implica uma organização curricular integrada, oposta à fragmentação identificada na organização do ensino no município em análise, o que pode justificar os resultados não satisfatórios na aprendizagem e, ao mesmo tempo, apontar possíveis caminhos pedagógicos para a superação desse quadro em nível nacional.

PALAVRAS-CHAVE*: Teoria Histórico-Cultural. Sistema Elkonin-Davýdov. Organização do ensino. Aprendizagem de matemática. Conceito de grandeza.*

RESUMEN*: Los resultados de evaluaciones externas, como PISA, SAEB y SAEP, revelan el bajo nivel de competencia de los estudiantes brasileños en Matemáticas en la educación básica. Basado en el supuesto de que el aprendizaje es el resultado de las interacciones educativas en las que participan los estudiantes, dirigimos nuestra atención a la forma en que se prevé la enseñanza de conceptos matemáticos. Para tener datos más objetivos sobre lo que se aprende y lo que se enseña, fue necesario elegir una situación particular para analizar, en este caso, los datos de las evaluaciones externas de un municipio en Paraná, así como la organización curricular de esa disciplina en el municipio respectivo. Basado en estudios sobre la forma de organización de la enseñanza desde la perspectiva del Sistema Elkonin-Davýdov, la organización de la enseñanza matemática debe guiarse por y para el concepto de grandeza, alrededor del cual se articulan álgebra, geometría y aritmética. Esta articulación implica una organización curricular integrada, opuesta a la fragmentación identificada en la organización de la enseñanza en el municipio bajo análisis, que puede justificar los resultados insatisfactorios en el aprendizaje y, al mismo tiempo, señalar posibles caminos pedagógicos para superar esta situación a nivel nacional.*

PALABRAS CLAVE*: Teoría Histórico-Cultural. Sistema Elkonin-Davýdov. Organización de la enseñanza. Aprendizaje de matemáticas. Concepto de grandeza.*

Introduction

Mathematical knowledge is essential in all areas of life and is part of it as soon as relationships between people and objects are established. Interpersonal relationships and activities performed by human beings give rise to logical-mathematical knowledge, as well as to other knowledge. So, from a very early age, the subject thinks mathematically about various everyday situations, even without being aware of this mental action (CARAÇA, 1951). Since childhood, the subject organizes, separates, compares and classifies objects, shares the chocolate bar with his brother, earns coins and gathers them in the piggy bank, loses pieces of his riding game and notices the lack of them, is unhappy if he receives the smaller part of a cake, that is, it acts constantly with mathematical concepts.

It seems to us that so far we have not encountered any problems with Mathematics. However, when it becomes a school subject, difficulties arise and we start to hear, whether in the school environment or outside, phrases such as 'I don't like math', 'Mathematics is difficult', 'this thing with numbers is not for me'. The science of Mathematics is considered, by both teachers and students, a challenge to be faced and unraveled as if it were a 'black box' of difficult access. We understand such concerns and fears, especially when related to the history of failure on the part of those who teach and those who learn, the high rates of recovering students, failure and dissatisfaction with the process and its outcome.

In this context, Mathematics retains a bad reputation, igniting discussions that often blame the student: 'the child does not have logical reasoning', 'he does not carefully read the problem-situation and/or the statements of school assignments', 'the child takes all the numbers that appear in the problem', and so on. Unfortunately, as Talizina (2000) reiterates, many teachers and mathematicians do little in this situation because they are

> [...] advocates of the genetic nature of mathematical capabilities. Thus, teachers often explain a student's poor math grades as a lack of math skills. In addition, they add that this student's parents were also not successful in math. [...] and they do not consider that their formation during the process of studying mathematics is possible. In this case, the teacher is practically not responsible for the results achieved by the students. (TALIZINA, 2000, p. 17, our translation).

Silveira (2002) observes, together with the bad reputation of Mathematics, the effect of accepting the failure of learning in this school subject revealed in the speech that 'Mathematics is difficult' and, therefore, it is 'for a few'. Low grades and/or failure in this subject are seen naturally by the school community, which, therefore, corroborates the acceptance, trivialization and reproduction of this phenomenon, often without questioning it. Thus, positioning and discourse “interfere in the relationship between the subject who teaches and the subject who learns” and the latter ends up suffering the negative effects of this relationship (SILVEIRA, 2002, p. 6, our translation).

The relevance of mathematical knowledge, for the subjects, is explicit in the social relations in which they are inserted, as well as the difficulty they encounter in using them, and we cannot naturalize this phenomenon. To denaturalize it, we need to understand it, thus, the objective of this text is to bring to analysis and reflection elements that integrate the movement of this phenomenon. As a starting point, we explain the level of proficiency in Mathematics of students at the national level, as well as students from a municipality in Paraná, in order to know the school performance in this area of knowledge. Next, we discuss the epistemological plan of Mathematical Science and the organization of teaching corresponding to this plan, from the perspective of the Elkonin-Davýdov System, in order to reflect on its possible contributions to the organization of the teaching and learning mathematical processes of concepts. Finally, basing ourselves on this teaching system, we analyzed the singular-particular mode of curricular organization of Mathematics in that municipality, as in the singularities general aspects of Mathematics teaching are manifested, revealing its deficiencies.

What do you learn in math classes?

The difficulties found in the teaching and learning processes are undisputed: they are present in everyday school life, in the practice and discourse of those involved, in internal and external assessments. One way to identify student performance in Mathematics was to analyze the results of external assessments. For this, we carried out a survey of data from PISA, Prova Brasil and SAEP. From the last two evaluations, we extracted data corresponding to the results of the proficiency of students who are in the 5th and 6th grades, respectively, to verify the Mathematics learning of those who complete the first stage of elementary school. Even recognizing the weaknesses of external evaluations, with their accountability policy, ranking of schools and regulation of the distribution of funds to these institutions, we also understand that, not being used as an isolated instrument for the elaboration of public policies, they can collaborate, along with other evaluative instruments, to map student learning, to detect problems and plan possible solutions.

The *Programme for International Student Assessment* (PISA) -is responsible for carrying out comparative assessment, applied to students aged 15 years, age at which the completion of compulsory basic education is assumed in most countries (BRASIL, 2015). PISA assesses students' proficiency in Science, Mathematics and Reading every three years. PISA-2012 brought together 65 countries and some independent territories, such as Hong Kong, Macao, Shanghai and Taiwan. In this edition, Brazil ranked 58th in Mathematics, 55th in Reading and 59th in Science. In PISA-2015, the ranking results show that Brazil was ranked 66th in Mathematics, 59th in reading and 63rd in Science (BRASIL, 2015; 2016). The PISA-2018 results published in 2019 indicate that Brazil ranked in the 69-72 range of the ranking, considering all participating countries/economies[4]. The results of the last three editions of PISA show a drop in the level of proficiency of students in the subject of Mathematics, in the final stage of basic education.

[4] PISA-2018 results. Available: http://download.inep.gov.br/acoes_internacionais/pisa/documentos/2019/relatorio_PISA_2018_preliminar.pdf. Access: 10 May 2020.

The Prova Brasil, introduced in 2005, is held every two years and, since the first edition, has undergone restructuring. Until the last edition, Portuguese Language and Mathematics tests were applied to students in the 5th and 9th years of public and urban schools (BRASIL, 2014). In 2015, the municipality of Paraná, taken as a singular-particular situation of analysis, obtained an average proficiency of 263.09 in the Mathematics test, which corresponds to level 6 (250 to 275), on a scale ranging from 0 to 10 (below 125 to 375), in their public schools[5]. In 2018, the results of the 2017 Prova Brasil were released, which revealed a stagnation in the proficiency indices in Mathematics of students in the 5th year of public schools, with an average grade of 263.66.

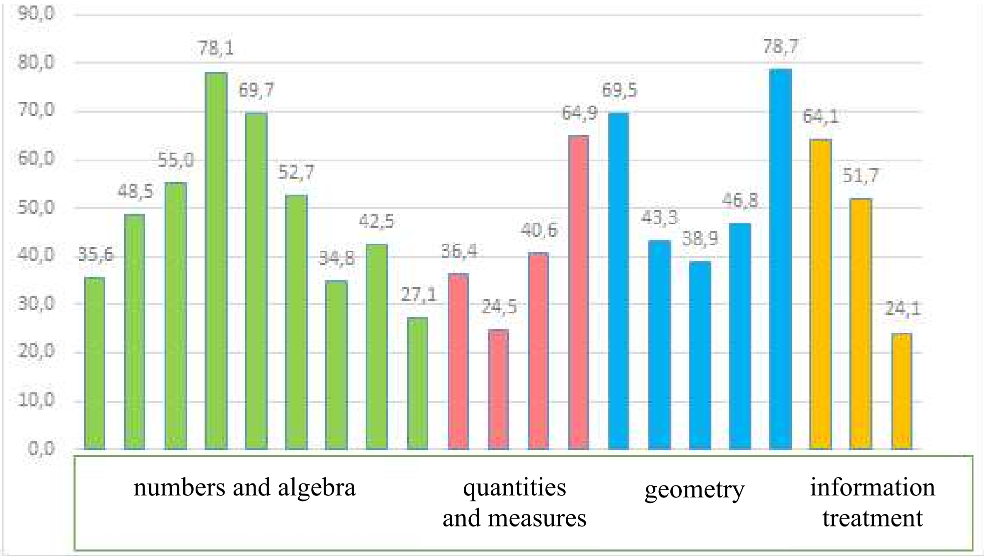
In 2012, the Paraná Basic Education Assessment System (SAEP) was created, which conducts tests in Portuguese and Mathematics. Four SAEP applications have already been carried out; in the first, in 2012, students from the 9th year of elementary school and the 3rd/4th year of high school were evaluated. In the second, in 2013, students from the 6th year of elementary school and the 1st year of high school were evaluated. In the evaluation carried out with 6th grade students, the municipality in Paraná under analysis reached an average proficiency of 212.3 in Mathematics. It fits, therefore, in level 4 (from 200 to 225 points), called basic level, considering a scale from 0 to 500 (PARANÁ, 2013). In 2017, students in the 9th grade of elementary school and 3rd and 4th grades of high school were evaluated. In the 2018 edition, the program evaluated students from the 6th year of elementary school, the 1st grade of high school and Youth and Adult Education. The 6th year students in the municipality of Paraná, in question, remain at the basic level of learning and the average proficiency grade, in Paraná, is 226.4[6].

Next, we present the table and graph of the results obtained by students in the 6th year of elementary school, in the municipality in question, in 2013[7]. The analysis of this material shows that the theme quantities and measures, with four descriptors, had the lowest average percentage: 41.6% of correct answers. The topic of information treatment was left with 46.63%; geometry, 55.44%; numbers and algebra, 49.33% (PARANÁ, 2013). Therefore, the general average of the percentages reached 48.25% of correct answers for all the content evaluated in this test. Thus, if we consider the results measured by this evaluation instrument, we could say

[5] SAEB-2015 and 2017 results. Available: http://portal.inep.gov.br/educacao-basica/saeb/resultados. Access: 10 May 2020.
[6] SAEP-2018 results. Available: http://www.saep.caedufjf.net/wp-content/uploads/2018/11/PR-SAEP-2018-RP-LP-WEB.pdf. Access: 10 May 2020.
[7] The detailed results, as shown in the 2013 chart, are of restricted access and we only obtained those from 2013.

that students complete the initial years of elementary school with "mastery" of approximately half of the concepts foreseen for this stage.

Figure 1 – Percentage of correct answers by descriptor - Mathematics - 6th grade ES: 2013



Source: Regional Education Center of the Municipality of Paraná

It may seem that the descriptors – specific objectives of each topic – are independent of each other and that the generating factor of low performance in each one of them is, therefore, distinct. However, first, it is necessary to consider that the concepts take place in a system of concepts and the formation of one intervenes in the formation of the other. As Vygotski (1931) proved in his experiments, there is a reciprocity in the relationship and a transference of concepts within a system, which are a reflection of the reciprocity of the phenomena in reality. This belonging and this reciprocity turn the concept into an important mediating instrument for knowledge in the real world and for the assimilation of humanity's social experience. Second, it is necessary to consider that, in the system of mathematical concepts, as Davýdov (1982) admits, magnitude is the core concept of Mathematics and permeates all other singular mathematical concepts (real number, geometry and algebra), which are aspects of this object nuclear. The graph, however, makes explicit an assessment that emphasizes numbers and operations, unique concepts linked to the recording and control of measures of magnitude, a reflection of the same emphasis given to the organization of the Mathematics curriculum (a topic that will be discussed later on).

Having made these introductory considerations on the results of school performance in the field of Mathematics, which are low throughout basic education, we bring to the discussion

Mathematics science and the school organization of its teaching from the perspective of the Elkonin-Davýdov System, envisioning possible contributions from theory to learning mathematical concepts.

Mathematical science and the organization of its teaching from the perspective of the Elkonin-Davýdov System

Science, says Caraça (1951), is a system of knowledge that explains phenomena by general laws, which involve other interconnected conceptual systems, with its own genesis, its contradictions, its movements. Its purpose is not only to describe and define phenomena as something ready and definitive, it is not a static system of terminology. Kopnin (1978) clarifies that knowledge of any science does not arise from the unknown, nor is it finished and, for this nature, knowledge must consider the historical character of the studied object, which implies a logic in its origin according to human needs.

Mathematics, like other sciences, arises from the needs of man and is the result of observations, studies, investigations, through which one seeks to understand phenomena and dominate nature. Thus, the greater the knowledge about a phenomenon, the greater the possibility of predicting it, causing it and/or controlling it.

However, in school programs, observes Ilyenkov (2007), there are too many absolute truths, which are definitively established for students to 'swallow' them without knowing the movement of knowledge and its totality. They are not prepared to actively seek answers to the questions posed by life or school, nor to the contradictions that require intellectual work. Thus, ready knowledge, as given by the school,

> [...] without the road that leads to it is a dead [...] bone corpse, the skeleton of truth, incapable of independent movement. [...] An established scientific truth, recorded in verbal terminology and separated from the route by which it was acquired, becomes a verbal shell, even if it contains all the external signs of "truth" (ILYENKOV, 2007, p. 21, our translation).

The author emphasizes, however, that he maintains the hope of a didactic reconstruction that overcomes the conservative view of teaching that, based on verbal terminological manipulation, 'hammeres' in the student's mind the abstract under the guise of concrete, mistakenly understood as what it is obvious, visible, empirical. His hope finds support in the didactic research of D. B. Elkonin and V. V. Davýdov, developed in the laboratory of the Institute of Psychology of the Academy of Pedagogical Sciences of the Russian Soviet Federative Socialist Republic (RSFSR), which has the Historical-Cultural Theory as its foundation. Elkonin and Davýdov's didactic proposal is based on the logical-dialectical understanding of thought and on the connections between the universal and the singular, between the abstract and the concrete, between the logical and the historical.

In the Elkonin-Davýdov System, the organization of the study, as explained by Ilyenkov (2007), the student, from the early years of schooling, must assimilate scientific knowledge, reproducing (in short) the real process of genesis and development of this knowledge socially produced throughout history. It does not mean that he has to reinvent the conquests already made by humanity, as this is unnecessary: the idea is that he redoes the logic of the path taken, which carries, in itself, the historical aspect (logical-historical character). In this way, he appropriates the concepts and formulas as a co-participating subject in the creative process instead of memorizing them to later reproduce them in new tasks and evaluations. Thus, the concepts become, for the subject, general principles with a real concrete character (abstract-concrete relationship) that will be used by him in the resolution of other particular tasks or specific real situations (universal-particular relationship), therefore, these concepts begin to present meaning and sense.

The school, says Davýdov (1982), needs to overcome the empiricist understanding of object knowledge and ensure students the possibility of making abstractions, generalizations and mastering theoretical concepts in their genesis and essence, from the beginning of schooling. The generalization and formation of theoretical concepts imply, therefore, the abstraction of the essential aspects of the object in its logical-historical origin. In this sense, school subjects, according to this author's proposal, are not composed of a list of definitions and illustrations, but of systems of concepts that have their core concept as their axis.

According to the studies by Davýdov (1982), the core concept of Mathematics is the concept of magnitude because this is the genetic foundation of the real number and, consequently, is the determinant of the emergence of other numbers: natural, integer, rational and irrational, so as well as the relationship between them. Its genesis is, says Caraça (1951), in the activities performed by the subjects, in the most varied circumstances; for example, men had the need to measure the extension of land, establish the value of taxes, the volume of a liquid to market, the right amount of seeds to sow on a certain land. That is, this knowledge was produced as a result of the need to know different quantities and numerically control their quantitative variation.

From the general concept of greatness are deduced the particular concepts of number, which constitute its manifestation. Based on these relations between the general and the particular, the author concludes that magnitude becomes the core concept in the formation process of theoretical thought in Mathematics. Numbers, in turn, are "a singular and particular case of representation of the general relations between quantities, when one of them is taken as a measure for calculating the other" (DAVÝDOV, 1982, p. 434, our translation). In this way, number becomes a quantitative characteristic of magnitude.

Measurements of a quantity also establish relationships with geometry, arithmetic, and algebra. For example, when you want to calculate the contour measurements of a polygon and your arithmetic calculation is not expressed only in numbers. To measure a certain quantity, certain values can be represented in a generic way, using letters and symbols. We observe, then, that there is an interconnection of algebraic, geometric, arithmetic meanings with the concepts of magnitude. Such interconnection cannot be disregarded in the organization of teaching Mathematics at school.

However, not infrequently, mathematical concepts are taught without considering the connections between them, as well as their connections with the real world; is taught how to count and measure objects without revealing their internal properties and their relationships under given conditions. As Ilyenkov (2007) exemplifies, countless students pass as incapable because the try to add kilograms to meters. Why do they do it? Because the first mathematical concepts developed at school are linked to counting anything with the natural number: apples, animals, people, wooden slats, children, iron dumbbells, water bottles, in short, anything singular perceived by the sensory. So, the student does not observe the abstract qualities of the object: its mass, length, capacity, among others, but the pure quantity, as a function of the natural number they were taught to verbalize in memory counts, without understanding the conceptual essence.

The organization of teaching of concepts of magnitude presupposes the proposition of conditions and actions necessary for them to be formed by the student, under the teacher's intentional guidance. It is necessary to act in a way that reveals the genesis and connections of scientific concepts so that the student understands the internal aspects of the object, relating them with its external aspects. This process includes levels of generalization of the concept that are developed through the scientific language of the teacher, which guides the student in associating the abstract and empirical characteristics of the object with each other and among other concepts, thus constituting a system of concepts. This language creates the genetic conditions that guide and drive the process of generalizing scientific concepts and developing theoretical thinking.

Based on the Elkonin-Davýdov System, we advance in the analysis of the singular-particular mode of curricular organization of Mathematics in the municipality of Paraná, revealing general aspects of Mathematics teaching. Analyzes that allow the identification of gaps and deficits in the program of this discipline, which contribute to the results of the students' learning process.

What is taught in Mathematics classes?

In this section, we analyze the curricular organization proposed for the early years of elementary school in a municipality in Paraná. The focus is on the contents and objectives of teaching Mathematics as a way of representing what guides the *modus operandi* of the teacher's action in this area of knowledge. For this, we analyzed the contents and objectives of the magnitudes and measures axis of the general curriculum of the subject of Mathematics from the 1st to the 5th year and of the specific curriculum of the 4th year. Both curricula were prepared by the pedagogical team of the Municipal Education Department and, here, we synthesize them to make them more explicit. We seek to demarcate the path proposed to teachers and students in the teaching and learning processes of mathematical concepts. Such path can express possible theoretical-methodological causes of the results of the assessments that students in this municipality obtained in this area of knowledge, as shown in the first section. Considering that there are multiple determinations for the phenomena, we do not neglect the fact that these results are also related to extra-school factors present in the social, political, economic, cultural context, but the discussion here permeates the theoretical-methodological aspects of school processes.

The curriculum structure of the early years of elementary school, the school level for which our discussion is aimed, organizes the contents into curricular components: Art, Science, Physical Education, Religious Education, Geography, History, Mathematics and Portuguese and English Language. These components are subdivided into teaching axes, structuring contents, specific contents and specific objectives. Mathematics comprises the axes 'numbers and operations', 'quantities and measures', 'space and shape' and 'information treatment'. We observe that the general objective of each axis is repeated from the 1st to the 5th year, as can be seen in the table below (MUNICÍPIO PARANAENSE, 2012).

Table 1 – General curriculum of Mathematics from 1st to 5th year of elementary school

Axis	Structuring content	General objective
Numbers and operations	The concept of number and operations	Understand the historical construction of number as a human need, in order to know how men controlled their objects at a given time and how we represent and use numbers today.
Quantities and measures	Measures of time/mass/length/ capacity/value	Recognize measurements and perform estimates and measurements with standard and non-standard objects, in order to use the measurements in different situations of daily life.
Geometry	Geometric shapes and spatial location	Identify geometric shapes through their characteristics and paths, through drawings, representation schemes and orality, in order to use this knowledge to recognize objects in space and locate themselves in the environment where they live.
Information processing	Graphs, Tables and Lists	Identify information contained in lists, graphs and tables, in order to know how to represent this information in daily routines and read this information present in various texts conveyed in social context.

Source: Município Paranaense (2012)

As in the external evaluations discussed above, in the general curriculum of Mathematics (Table 1), we find the fragmentation of knowledge in this science, which is not exclusive to it, as it is present in other areas of school knowledge, at different levels of education and at the national level. Vygotski (1982) understands that this conceptual secession hinders the appropriation of scientific concepts, as awareness of concepts occurs within a system through the recognition of certain relationships between them, which is compromised when working with concepts is done in a piecemeal way.

We still have the vertical exposure of the axes, which denotes their hierarchical character and makes the interrelationship between them difficult. When considering the general objectives, we observe the emphasis given to the number and operations axis (arithmetic) to the detriment of the other axes, as the student is expected to 'understand' the concept, a verb that announces cognitive domain. While, in the other axes, the student is expected to 'recognize' or 'identify' the concepts related to quantities and measures, geometry and information processing. The actions of recognizing or identifying do not allow the analysis of the internal and essential aspects of the scientific concept studied, which are crucial to the development of theoretical thinking, that is, generalizing thinking, which allows conscious actions with the concept. Rather, they are actions with external and particular aspects of the concept, leading to the development of empirical thought characterized by isolated judgments and restricted practices.

To get even closer to the organization of the teaching of Mathematics proposed in this municipality, and to understand what is taught in the classroom, we analyze the specific contents and objectives of the axis (structuring content) quantities and measures of the 4th year, so that it becomes more explicit the way of curricular organization of this discipline.

Table 2 – Contents and specific objectives of the 4th year quantities and measures axis

MATHEMATICS CURRICULUM - 4th YEAR Structuring contents: measures of time / mass / length / capacity / value	
Specific Contents	Specific objectives
Measure of time	
Calendar: year, decade, century and millennium	Identify and relate millennium, century, decade and year, to temporally locate in different situations that involve the reading of these data.
Full hour, half an hour, minutes and seconds	Reading and recording hours (on hand and digital clocks) as well as solving significant problem situations involving the interval and the fractionation of time to recognize their social use.
Measure of length	
Meter, half meter, decimeter, centimeter, millimeter and km	Recognize the decimeter, centimeter and millimeter as a fraction of a meter to realize the importance of this fractionation in various daily situations. Identify the Km as a multiple of the meter in order to relate these measurements
Measure of mass	
kilo, half kilo and gram	Recognize the gram as a fraction of a kilo in weight comparison activities, in order to understand the importance of this fractionation, in various situations of our daily lives.
Measure of capacity	
liter, half liter, milliliter	Recognize the milliliter as a fraction of a liter in pouring activities (composition and decomposition of the liter), in order to understand the importance of this fractionation, in various situations of our daily lives.
Measure of surface	
Area and perimeter	Recognize the notions of area as a measure of surface, and of perimeter as a contour measure, understanding the idea of area as a multiplication and perimeter as an addition, so that the student can perform the calculation of these measures.
Comparison of perimeter and areas of two figures	Compare the area and perimeter of two or more figures, recognizing the relationships that are established between them when we enlarge or reduce these figures.
Measure of time, length, mass, capacity, temperature, velocity and surface	
Measurement Instruments	Recognize and use the various measuring instruments existing in our daily lives, in order to understand the role of each one in carrying out activities in their context.
Measure of value	
Bills and coins	Identify bills and coins from our monetary system and understand that having more bills or coins does not imply having more money; use this knowledge in purchasing situations; Perform the composition and decomposition of bills and coins, in order to verify the use of these procedures in our lives; Solve situations that demand the use of bills and coins, identifying which strategies used by the market are beneficial or not for consumers.

Source: Município Paranaense (2012)

In the 4th year curriculum, we observe the same conceptual sections as the general mathematics curriculum, in which the types of magnitude and their respective measures are separated. It is not possible to infer any link between the quantities and their relationship with geometric and arithmetic concepts, much less with algebra, which is not included in the curriculum.

It is also possible to infer that the practice suggested by this curricular proposal includes actions that are closer to empiricism. These actions are evidenced in the specific objectives that revolve, mostly, around 'identifying', 'recognizing' and 'comparing' quantities, actions that tend to remain in the external aspects of objects, in what is perceived by the senses.

What knowledge do they acquire and what kind of thinking do students form that only 'identify' characteristics of geometric shapes, 'represent' the identified information, 'recognize' the decimeter, centimeter and millimeter as a fraction of a meter and 'recognize' the notions of area and perform the calculation? We understand that these very particular and empirical actions, as well as the essentially abstract ones, do not allow the student to know the movement, the essence, the internal and external connections, the logic and the origin of the concepts, which form the necessary theoretical thinking for those subjects act consciously in reality.

Another aspect observed is the emphasis on actions related to the student's immediate needs, such as: 'carrying out the composition and decomposition of bills and coins, to verify the use of these procedures in our lives'; 'recognize and use the various measuring instruments existing in our daily lives'. So, the objectives follow, ending with the intention of recognizing the concepts 'in our everyday situations'. According to Kosik (1976), the main objective that should guide the organization of teaching at school is the appropriation of knowledge that enables the student to understand not only everyday phenomena, but also those that are beyond the limits of the immediate context, of actions empirical and particular.

Ilyenkov (2007) clarifies that the problem of learning Mathematics lies in the didactic organization of its teaching, as misconceptions predominate in it "[...] about the relationship between the 'abstract and the concrete', the 'general and the singular', of 'quality and quantity' and the thought formed about the world perceived by the senses that, until today, unfortunately, is found at the base of many didactic programs" (ILYENKOV, 2007, p. 41, our translation). Such programs, according to the author's analysis, do not teach the student to think 'concretely', as they confuse concrete with empirical. To think concretely is to understand a given phenomenon in its totality and to think empirically is to perceive only the particularities that are most evident to them. These programs favor teaching procedures that are sometimes too theoretical, with proposals for memorizing definitions and formulas, without understanding them; now strictly supported by empirical actions, which are self-contained, without developing the necessary abstractions and theoretical generalizations.

Final considerations

The general laws of a science and the knowledge produced by it, like mathematical science, have as their primary source the human needs placed in objective reality, which manifest their totality and interconnections. Admitting such general laws, school education, with its curriculum, needs to overcome its fragmented and empirical characteristic, as we observed in the discussions held in the previous subsections, in favor of an education that considers the movements of reality and its genesis, as well as the understanding the movement of knowledge and thinking about reality.

Mathematical knowledge, despite being essential to human activities and to understanding the reality that surrounds us, is still not accessible to most Brazilian students, even after completing basic education, as verified in external evaluations. Outlining the construction of paths, in the didactic-pedagogical scope, to overcome the learning difficulties of Mathematics implies identifying and understanding them, having the curricular organization of the discipline as a background. Since is from there that the teaching, learning and development processes derive.

The analysis of the curriculum of the Mathematics discipline revealed a model of organization fragmented into axes and structuring contents, which do not lead to study actions that enable the interconnection of mathematical conceptual systems, the understanding of their genesis and movements. In the learning objectives, we identified explanations of actions or operations with concepts at an empirical level, which lead to the perception of the external particularities of the object of study related to the student's daily life, therefore, they do not corroborate the theoretical abstractions and generalizations of what is essential.

In view of the exposed weaknesses, we understand that the proposal of teaching organization, from the perspective of the Elkonin-Davýdov System, can guide reflections and qualitative restructuring in the school curriculum. For this, it is essential to know the epistemological plan of the science to be studied in a school environment, to understand its genesis and structure, in order to organize the corresponding study procedures. This requires the investigation of the contents that make up the disciplinary program and the appropriate didactic directions to teach them.

In the Elkonin-Davýdov System, the structural content, the axis of the school discipline, is composed of scientific concepts and their corresponding mental actions. The teaching procedure, consistent with this structure, which can be unique in each discipline, must be based on the core concept of science, in the case of Mathematics, on the concept of quantity. Actions with this concept are carried out in such a way that the student reproduces their logical-historical character, putting the general-particular and abstract-concrete relations into dialectical movement. This didactic proposal also implies study actions with systems of interconnected concepts that reveal the essence of the object of study, which lead to the formation of abstractions and theoretical generalizations. This education system aims to overcome empirical thinking, by incorporation, and the development of theoretical thinking.

From the above, we understand that the proper organization of the teaching of Mathematics is associated with the study of its nuclear and general object, that is, the study of the concept of magnitude, from which it is possible to extract, relate and understand the other mathematical concepts of algebra, geometry and arithmetic.

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