CONNECTION BETWEEN PROFESSIONAL DIDACTICS AND DEVELOPMENT DIDACTIC ENGINEERING: A DISCUSSION FOR THE FORMATION OF MATHEMATICS TEACHERS

RELAÇÃO ENTRE A DIDÁTICA PROFISSIONAL E A ENGENHARIA DIDÁTICA DE DESENVOLVIMENTO: UMA DISCUSSÃO PARA A FORMAÇÃO DE PROFESSORES DE MATEMÁTICA

RELACIÓN ENTRE DIDÁCTICA PROFESIONAL E INGENIERÍA DIDÁCTICA DEL DESARROLLO: UNA DISCUSIÓN PARA LA FORMACIÓN DE PROFESORES DE MATEMÁTICAS

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ABSTRACT: This work aims to lead the discussion between Didactic Development Engineering, an important French methodology used for four decades in French education arising from Didactics of Mathematics, along with Professional Didactics, a francophone strand that builds professional learning, aiming build devices for the initial or continuing education of teachers, of particular interest, mathematics teachers. The discussion is based on the point of view of Chevallard (2009), Pastré, Mayen and Vergnaud (2006), Pastré (2008) and Alves (2018a; 2018b; 2019; 2020). The synthesis of the analyzed researches shows that Didactics of Mathematics and Professional Didactics, maintains a tenuous line in what concerns the formation of teachers.

KEYWORDS: Professional didactics. Development didactic engineering. Teacher training.

RESUMO: *Este trabalho tem por objetivo levar a discussão entre a Engenharia Didática de Desenvolvimento, uma importante metodologia francesa usada por quatro décadas na educação francesa advinda da Didática da Matemática, juntamente com a Didática Profissional, uma vertente francófona que constrói a aprendizagem profissional, visando construir dispositivos para a formação inicial ou continuada de professores, de interesse particular, professores de Matemática. A discussão se baseia no ponto de vista de Chevallard (2009), Pastré, Mayen e Vergnaud (2006), Pastré (2008) e Alves (2018a; 2018b; 2019; 2020). A síntese das pesquisas analisadas mostra que a Didática da Matemática e a Didática Profissional mantém uma linha tênue no que concerne à formação de professores.*

PALAVRAS-CHAVE: *Didática profissional. Engenharia didática de desenvolvimento. Formação de professores.*

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***RESUMEN**: Este trabajo tiene como objetivo liderar la discusión entre la Ingeniería del Desarrollo Didáctico, una importante metodología francesa utilizada durante cuatro décadas en la educación francesa que surge de la Didáctica de las Matemáticas, junto con la Didáctica Profesional, una vertiente francófona que construye el aprendizaje profesional, con el objetivo de construir dispositivos para el inicial. o formación continua de profesores, de especial interés, profesores de matemáticas. La discusión se basa en el punto de vista de Chevallard (2009), Pastré, Mayen y Vergnaud (2006), Pastré (2008) y Alves (2018a; 2018b; 2019; 2020). La síntesis de las investigaciones analizadas muestra que Didáctica de las Matemáticas y Didáctica Profesional, mantiene una tenue línea en lo que concierne a la formación de profesores.*

***PALABRAS CLAVE**: Didáctica profesional. Ingeniería didáctica del desarrollo. Formación de profesores.*

Introduction

Through investigative textbooks on Didactics of French origin, we will initially understand Didactic Engineering and its evolution in the field of Education and Teaching, and how Professional Didactics has currently taken up space regarding the professional formation of teachers; in particular, we take the Mathematics professional.

In Brazil, the theories that make up the Didactics of Mathematics have been adapted in Mathematics Education used in teacher education (ALVES, 2011). Theories emerged in the 1970s and 1980s, considering the teaching-learning-formation, defined as: Didactic Engineering, Theory of Didactic Situations, Theory of Conceptual Fields.

In France, after changes in the Taylorist pedagogy of work, the Delors law guaranteed a better qualification in formation for several professionals, including teachers; however, Didactic Engineering and the Theory of Didactic Situations in the 1990s did not guarantee the professional qualification of teachers.

Chevallard (2009) comments that Didactic Engineering started to be used in another way, as proposed by Brousseau (1986), fitting the need to use Professional Didactic Engineering[3] for teacher education, and its use in formative environments is urgent.

Therefore, assuming that French educational theories have contributed a lot to the teacher formation framework, we observe that Didactic Engineering reformulated its formative paradigms at the French Summer School in 2009, turning to the triad: formation-research-teaching (PERRIN-GLORIAN, 2011; TEMPIER, 2016). At the same time,

[3] With the same meaning Professional Didactics.

Professional Didactics has brought excellent observations and explanations about the context of on-the-job formation (PASTRÉ; MAYEN; VERGNAUD, 2006; PASTRÉ, 2011).

The article presents a historical and complementary itinerary between Didactic Engineering in the context of its evolution during almost four decades of existence, and a relationship with Professional Didactics, as it presents a complete theoretical framework for the teacher to understand the *modus operandi* in professional didactic situations, according to the point of view of Chevallard (2009), Pastré, Mayen and Vergnaud (2006), Pastré (2008), Alves (2018a; 2018b; 2019; 2020).

The synthesis of the analyzed research shows that the Didactics of Mathematics and the Professional Didactics maintain a fine line in what concerns the formation of teachers. After a series of evolutions, Didactic Engineering indicates a second level to study the adaptability of situations to regular teaching, aimed at the formation of teachers who teach mathematics (TEMPIER, 2016), defined as Didactic Development Engineering; similarly, Professional Didactics has recently focused on teacher education using the assumptions of French ergonomics, in the analysis of work (PASTRÉ, 2011).

Didactics of Mathematics in the context of teacher education

In the 1920s, the teaching of Mathematics was taken over by the Bourbaki movement[4], which considered the dissemination of set theory in mathematics teaching curricula. Later, after World War II, there was a deepening of scientific knowledge, coming to reformulate the mathematics curriculum (LOPES, 1994).

Thus, the 1960s and 1960s were marked by the Movement of Modern Mathematics (MMM), having as its main characteristic the approach to the concepts of abstract mathematics, such as: axiomatic thinking, high degree of abstraction, greater logical rigor, deductive method, among others, maintaining a strong link with the structuralist model (PINTO; ALMEIDA; DINIZ, 2007).

On the other hand, Piaget (1975) made some comments about the MMM using as a model his psychogenetic theory, "[...] a child is not able to reason from pure hypotheses expressed verbally, and they need, in order to make a coherent deduction, to apply them to manipulable objects" (PIAGET, 1975, p. 15, our translation). Based on conceptions like these, the MMM obtained an outcome based on failure due to non-fulfillment of its objectives.

[4] Abbreviated from Nicolas Bourbaki, pseudonym for a group of French mathematicians: Cláudio Weil, Claude Chevalley, Henri Cartan, among others.

Given the changing scenario in the 1980s, Didactics of Disciplines (or Didactics of Sciences) emerged, a grouping of theories based on the relationship of Sciences (Mathematics, Chemistry, Biology and Physics). However, from the Center Observatory and Research in Teaching of Mathematics (COREM), in France, some important theories emerged through the Didactics of Mathematics (DM):

> Brousseau wanted to propose a theoretical model that would contribute to the mathematical learning of students (from different levels of education), understanding learning as a process of knowledge construction, in the light of Piagetian theory. A brief observation should be made here: although Piaget attaches importance to interactions, this is not the center of genetic psychology, on the other hand, TSD considers interactions between subjects fundamental and assigns a primary role to the teacher as a mediator of the learning process (BITTAR, 2017, p. 22, our translation).

As well as "[...] mathematics education in France was built on the recognition of the need to develop specific theoretical structures" (ARTIGUE, 2002, p. 60, our translation). In this context, in the early 1980s, three theories were already well developed in French Mathematics Didactics: Theory of Didactic Situations (BROUSSEAU, 1986), Theory of Conceptual Fields (VERGNAUD, 1990) and Didactic Engineering (ARTIGUE, 1991).

Considering such theories, we highlight Didactic Engineering (DE): the premise is that from the beginning it was engaged in the relationship between research and didactic action in educational systems and, as a development instrument, it started to attribute a role to research productions in relation to theoretical and practical knowledge. Therefore, DE maintained a preference among French educators during the 1980s (PERRIN-GLORIAN, 2011).

However, DE is called a research methodology, developed from four phases (or steps), consisting of: i) prior analysis (or preliminary analysis), ii) design and *a priori* analysis, iii) experimentation and iv) post analysis and internal/external validation.

In the first phase, it is understood by the prior analysis, which aims at a historical epistemology of mathematical content in the current molds of teaching, the students' conceptions and obstacles faced and evolutions in the face of obstacles, according to research objectives and the study of didactic transposition (ARTIGUE, 1998).

In the second phase, there is the *a priori* construction and analysis, characterized by the didactic variables that Artigue (1998) disposes as: i) *macrodidactics* and ii) *microdidactics*, ai) related to the global organization, ii) related to the organization of a session or phase. In addition, the *a priori* analysis should:

> Describe the choices of local variables and the characteristics of the didactic situation developed. • Analyze the importance of this situation for the student and in terms of the possibilities of actions and choices for the construction of strategies, decision-making, control and validation that the student will have. The student's actions are seen in the almost isolated functioning of the teacher, who, being the mediator in the process, organizes the learning situation to make the student responsible for their learning; • Predict possible behaviors and try to show how the analysis made allows controlling their meaning, ensuring that the expected behaviors, if and when they intervene, result from the development of knowledge aimed at learning (ALMOULOUD; COUTINHO, 2008, p. 67, our translation).

In the third phase, there is experimentation, the moment to put the entire device built into operation, and corrections can be made if necessary, regarding local analyses, implying a return to the previous phase (a priori analysis). In this phase, the data obtained during the realization of the teaching sessions in the respective productions of the students in the classroom or not are collected (ARTIGUE, 1998).

The last phase is characterized by *a posteriori* analysis and validation, as a result of the experimentation, in the exploration of the collected data to contribute to the didactic knowledge for the transmission of knowledge. Therefore, in this phase, it is understood that the result of the previous phase is used for data collection. Subsequently, from the analysis of the data, the validation process ceases, which can be of two types: internal validation and external validation. The internal validation is "[...] based on the confrontation between an *a priori* analysis and an *a posteriori* analysis" (ARTIGUE, 2002, p. 63, our translation), being opposite to the external validation, based on the comparison of the performance of experimental groups and class control (ARTIGUE, 2002).

On the other hand, in France, faced with a concern with qualified formation, French education was marked by quality guidance policies aimed at the formation of adults. In 1971, the Delors law was established in the field of professional formation for adults (CHAMPY-REMOUSSENARD, 2005).

> In France, the "adult formation" semantic network must date back to the 1960s; it was easy to speak, then, of permanent education. People who worked in this sector wanted to mark their specificity in relation to education in general and, more particularly, in relation to education. (BÔAS, 2012, p. 263, our translation).

The formation of adults was gradually constituted in the course of the 20th and 21st centuries; two circumstances prominently contributed to the formation of adults in France: the first is in the post-World War II period, in 1945, and later in the 70s, due to the evolution of

social relations, and the demand of the industrial field for formation with quality for the workers.

In the Taylor period it was common for competence to be linked to know-how; in other words, workers did not need to use intelligence in work situations, so, given the new context, there was a need for better formation for adults (employees) in the industrial sphere.

However, adult education and teacher education are not the same, as stated by Bôas (2012) when he says that the educational system is not aimed at activity, but knowledge, specifically in didactics, unlike formation of adults that considers the activity.

In this scenario of changes, in the 90's, the Professional Didactics (PD) emerged, a strand that intends to analyze the activity for the formation of professional competences. Pastré, Mayen and Vergnaud (2006) discuss that PD is supported by a theoretical and practical framework coming from ergonomic psychology, developmental psychology, discipline didactics and continuous professional formation.

Professional Didactics and Didactic Development Engineering

Professional Didactics (PD) was established after the Taylor-Ford period, having appeared for the first time by Pastré in 1992 in his thesis, being defined as a branch of professional formation for adults. Its epistemological foundations encompass a set of currents already mentioned above, but specifying:

- Conceptualization in action from developmental psychology, socio-constructivism and Conceptual Fields Theory (PIAGET, 1974; VYGOTSKI 1986; VERGNAUD, 1990);
- Ergonomic Psychology from Work Psychology, Work Analysis and French Ergonomics (FAVERGE, 1972; LEPLAT, 2006);
- Didactic of Disciplines (BROUSSEAU, 1986; VERGNAUD, 1990);
- Formation Engineering and Adult Training.

It is important to point out that the PD in the beginning considered the work activities of workers, and currently the PD has been extended to teaching activities. One of the works that explains more deeply about PD is "*La didactique professionnelle*" by Pastré, Mayen and Vergnaud (2006); in this work it is explained how this aspect emerged and the course of its development.

PD is the result of formation engineering and continuing education, consisting of a field of analyzed practices that gave rise to professional didactic engineering, a branch that uses work analysis to build formation devices aimed at professional competence.

PD is marked by the post-Taylorist period, in which professional formation started to be discussed in France, causing a technological innovation that demanded knowledge and experience from the workers. In this context, the contributions of ergonomic work analysis and cognitive psychology were relevant to the development of PD. Table 1 presents important concepts of this aspect.

Table 1 – Important elements that mark the PD

Notion or concept	Definition
Taylorism	Since Taylorism is about the organization of work (PASTRÉ, 2001).
Constructivism	Approach that affirms the individual's inherent capacity to build their knowledge through the conceptual restructuring of their accumulated experiences (VERGNAUD, 2000).
Activity	Any contextualized action that mobilizes the individual's socio-cognitive resources and can generate learning (LEPLAT, 1980).
Competency	Competence is the mobilization or activation of various knowledge, in a given situation and context. Competence is the ability to act in a situation. It is practiced in contexts (personal, social, professional). It concerns the performance of complex tasks (problem solving, decision making, project realization) (LE BOTERF, 2003).

Source: Devised by the authors

PD was born between these axes addressed, refounding on conceptualization in action, ergonomic psychology, didactics of disciplines and formation engineering. The conceptualization in action comes from the work of Vergnaud (1990), regarding the operative schemes used during professional situations. The work of the psychologist Leplat (2013) in distinguishing between the prescribed task and the actual activity was important for the subject to understand the work analysis.

The crucial point of emphasis of the PD is in its opposition to the didactics of the disciplines, specifically with the DM, based on the research of Brousseau (1986; 1998), which focused on the students, not covering the teacher's work activity.

Pastré (2008) states that TSD focuses on learning from academic knowledge and didactic situations, with PD having as its main objective professional work situations and

learning from work action, which he calls pragmatic knowledge, especially complex situations in which the worker moves an operative intelligence to excel, showing an ability that falls within his competence to manage unusual work situations.

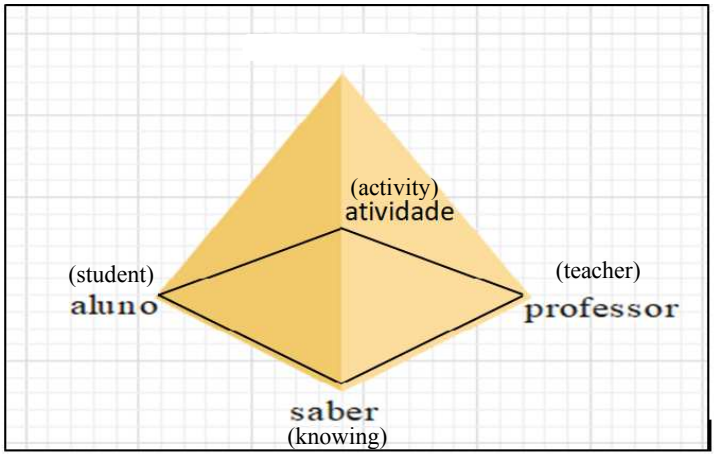
Pastré (1992) admits that before the creation of the PD, when he was still supervising the work of the miners in formation engineering, he always stopped at the work analysis, and this analysis was crucial for the creation of this trend. The author concluded that if the worker analyzes his activity, he will certainly improve his competence.

Initially, PD focused on the analysis of industrial work, however, in the last decade, it has been interested in analyzing the activity of other professions, including the activity of teaching teachers, due to the search for Science in Education in PD to understand the actual activity of teachers in initial or continuing education (PASTRÉ; MAYEN; VERGNAUD, 2006; VINATIER, 2013).

In Brazil, PD has a very recent picture in scientific research, however, the work of Alves (2018a; 2018b; 2019; 2020) has been a pioneer in the formation of mathematics teachers. This author has extended some TSD terms to the professional activity of Mathematics teachers; the works show a complement between the PD and the TSD, as well as some competence plans in face of three elements: students, co-workers and institutional norms.

Furthermore, Baudouin (1999) proposes a difference between the PD and TSD assumptions as shown in Figure 1: this author says that the didactic triangle (student-teacher-knowing) is insufficient to analyze the teacher's activity.

Figure 1 – Tetrahedron that displays the Professional Didactics for the analysis of the teacher's activity



Source: Adapted from Baudouin (1999)

Based on this three-dimensional figure, Baudouin (1999) concludes that the logic of scientific knowledge models such as TSD proposes is not sufficient to explain the unusual work situations that teachers tend to encounter in the professional environment. In light of this, Chevallard (2009) assumes that there was a negligence on the part of research by mathematics didacticians, "[...] we postponed the care of rethinking the didactic act, aimed at adults at work, with reference to the development of skills and experience at work" (CHEVALLARD, 2009, p. 9, our translation).

Chevallard (2009) says that after the explosion of adult formation in continuing professional formation at the end of the 1980s, DM had other research itineraries, "[...] the didactic engineering methodology is being used in different ways from that understood by Brousseau, including by some of the Didactics of Mathematics researchers, with the need to use Professional Didactics" (CHEVALLARD, 2009, p. 41, our translation), the scenario of formation and competence with a professional bias stood out in the face of PD.

Since the 1990s, DM has been thinking about formation strategies for teachers. But only in 2009, at the XV Summer School in Didactics of Mathematics (French, *École d'été de Didactique des Mathématiques*), held in France, did Perrin-Glorian make room for Didactic Development Engineering, aimed at training teachers (PERRIN-GLORIAN; BELLEMAIN, 2009).

In front of Perrin-Glorian (2011, p. 24, our translation): "[...] didactic engineering is not a finished object, but an object whose design must be able to continue in use", however, "[...] teacher education is a relatively new phenomenon in the community of researchers in didactics" (VERGNAUD, 1992, p. 23, our translation).

After a series of changes in the training of adults, DE took as its principle the formation of Mathematics teachers, being defined as Didactic Development Engineering. Perrin-Glorian (2009) concludes that there are three conditions for carrying out an DDE:

i) Leave certain freedom of action to the teacher: this condition is already valid at the first level, but now it is a question of defining the sequence of situations with the teacher and analyzing how he adapts the document provided to him.

ii) Using the documents produced, teachers should try not to reproduce the story, but the conditions of learning, the essential question for didactic engineering, being how to identify the essential elements for the effective performance of the activity.

iii) It is necessary to rely on first-generation didactic engineering that enables the construction of a fundamental situation and its analysis.

Even though some invariants remain, didactic engineering has become a valid theory for the community of researchers and professors who take teaching and learning as important variants for the classroom.

Teacher Professional Competence

The term competence began to be diffused in the late 1960s, being observed as a set of knowledge: know-how, scientific knowledge, knowledge in action (GILLET, 1991). In such a way, competence began to be established in industries in the 1970s in France, having a strong relationship with work performance; later, there was a structural break between the work and Taylor's pedagogy.

With this, the organization of work becomes the heart of competence, in addition, competence is intertwined with knowing how to act and react in a given work situation in which the individual is able to implement professional practices to solve problems arising from new work situations, as Pastré (2004, p. 217, our translation) defines:

> The skill of an operator, whether he is a machine operator or a vine grower, obviously does not come down to mastering the conceptual framework of the situation. But this is the key to the sense of competence, around which it will organize itself hierarchically according to abilities, procedures, management and resources etc. The identification of the conceptual structure of a class of professional situations corresponds to the cognitive dimension of skills.

PD takes the development of formation skills into account and is at the center of its concerns. We corroborate with Pastré (1999, p. 403, our translation), when saying that the PD epistemology "[...] comes within the scope of the production of educational resources containing work situations that support the formation and development of professional skills".

PD has principles parallel to behaviorism, in which competence (gathering of skills) was shown in the successful performance of a job function. In contrast, Pastré (2011) says that competence should not be measured in the face of good performance in the activity, but competence must show itself in a new work situation.

In addition, we take the following question: how does a worker take their professionalization? With this, Vergnaud (2001, p. 2) defines a competent professional in:

1 - A is more competent than B, if he knows how to do something B cannot do;

2 - A is more competent than B if he does it better. The best comparison presupposes complementary criteria (speed, reliability, economy etc.);

3 - A is more competent if it has a directory of alternative resources that allows the use of one procedure or another and thus more easily adapts to different scenarios that may arise.;

4 - A is more competent if it knows how to deal with a new situation, of a category that has never been found before.

However, it is not enough that a person has the resources (one might say "skills") to "be" competent: he must be able to organize and mobilize them in relevant combinations to manage "professional situations" leading to consideration of the related “performance criteria”. It is these “performance criteria” that will guide the construction of a combination of resources.

When the term competence invades the teacher's profession, it becomes evident that it is not easy to manage work situations in the classroom. However, Pastré, Mayen and Vergnaud (2006) point out that due to the search for answers, some researchers in Educational Science sought ergonomics to analyze the teacher's work.

Thus, we note that the framework of Professional Didactics lists contributions to the practice of teachers (Mathematics teachers), aimed at an evolution and development of skills at work.

In the classroom, the professional competence of the mathematics teacher is linked to the skills in everyday tasks carried out around the profession in the learning process at work. Therefore, Alves and Catarino (2019) assume the existence of three categories or levels, which manage the mathematics teacher's learning, through professional situations. Thus, the professional activities of the mathematics teacher must meet the level plans: "(i) classroom plan, (ii) workstation plan, (iii) the general plan of the educational institution, or school institution" (ALVES; CATARINO, 2019, p. 115, our translation).

Plan (i) (classroom) analyzes the teaching activity with multiple purposes, embedded in each other. A goal that the teacher must have is to be calm in his class (the subject) so that he can put the students to work; next, the teacher must provoke learning, ending with the cognitive development of his students.

In plan (ii), we present the teacher's professional competence through the political field and class social negotiation. At this level, in a detailed way, we seek to observe the evolution of the mathematics teacher, specifically at work. Pastré (1999) reminds us that the assessment of an individual in the workplace does not take place before a qualification system. Thus, the teacher's work should not be evaluated in the prescribed task, but the work performed in the institution.

In plan (iii) competence is linked to the cognitive mode. From the socio-constructivist models and the accentuated constructivism in the conceptual fields, the competence of adults is viewed in the workplace, starting to learn in the locus of work. Thus, "[...] the professional competence of the teacher is substantiated by a progressive transition and the coordination of activities of action and anticipation, construction and reconstruction of cognitive schemes, hierarchically structured by operational invariants" (ALVES; JUCÁ, 2019, p. 12, our translation).

We know that conceptualization precedes action, for this reason, cognition comes before we operate any actions. From this Piagetian assumption, we realize that competence develops the teacher's learning at work. Competence is a process of acquiring knowledge and professional action: the individual uses knowledge to improve their competence. We have that the fundamental and structuring nucleus of a trade is in the way of learning in the situations that the work develops during the worker's action.

Therefore, the teacher learns through practice at work, acting through a repertoire of actions typical of their work position, described by a regime of normative and official manuals that control the action. Thus, competence will be organized by a set of skills, emphatic in rules that are not always explicit, but detailed and pragmatic.

In the professional activity of the mathematics teacher, pragmatic concepts are part of learning on the job. We know that competence is not only linked to epistemic knowledge but is at the heart of pragmatism. Pastré (2002) addresses the main characteristics of pragmatism at work, which also extends to the activity of the mathematics teacher.

Final considerations

DM has a long tradition in Mathematics Education, regarding teaching, learning and teacher formation, being a set of theories that lead to improvement in teacher formation and improvements in the conditions for teaching and student learning. However, we cannot neglect that TSD and DE do not provide a complete context regarding work situations, requiring a complementarity with the PD to assess the professional teaching activity of Mathematics teachers.

Given this space, the complementary textbooks between the DM and the PD are shown to be relevant in the context of teacher education, especially for Mathematics teachers. It is considered that the Francophone strands have expanded throughout Europe and America,

mainly in Brazil. DE and TSD are recurrent in scientific research in the part of Mathematics Education and Teaching, however, PD is still new in the Brazilian research scenario.

However, some works have stood out in national research in an innovative way regarding the complementarity between these aspects for teacher education in Brazil; it was clear that this relationship between theories is relevant for understanding the context of initial or continuing education in relation to professional competence, professional didactic situations, professional obstacles, among others that help the teacher to understand the work.

This work has, therefore, the potential to show how PD has now become urgent, along the lines of adult formation, given the framework of changes that since the 1990s has provided transformations and evolutions in terms of teacher education, as in the case of DE, which currently has proven to be an DDE for the education and formation of Mathematics teachers.

Finally, after a period of continuous professional formation and post-Taylorism, theorists point out that PD and DM, in the face of a trajectory of evolutions and modifications, lead to a potential for postgraduate courses at the master's and doctoral level in teacher education, as is the case of the French-speaking Universities (Québec, Lyon, Éspé, among others).

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