CONTRIBUTIONS OF VYGOTSKIAN THOUGHT TO MATHEMATICAL MODELING

CONTRIBUIÇÕES DO PENSAMENTO VYGOTSKIANO PARA A MODELAGEM MATEMÁTICA

CONTRIBUCIONES DEL PENSAMIENTO VYGOTSKIANO AL MODELAJE MATEMÁTICO

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ABSTRACT: The present study is a theoretical essay that analyzes possible contributions of Lev Vygotsky's ideas to the teaching of Mathematics, through modeling. For him, the historical-cultural context shapes the psychological element, determining the way of thinking. People from different cultures have different psychological profiles. Psychological functions are developed over time and mediated by social interaction, through cultural symbols. Language is related to culture and depends on social factors. Concepts are historically constructed and internalized in a particular way by individuals, in a broad, integrated, holistic and dynamic way. Such principles can contribute to a greater understanding of Mathematical Modeling, since it starts from situations presented with real problems, aligned with the students' experiences leading them to assume the protagonism in this process, as they develop a mathematical model, in an environment of learning in which the interaction is present.

KEYWORDS: Historical-cultural process. Psychological functions. Mathematical modeling.

RESUMO: *O presente estudo é um ensaio teórico que analisa possíveis contribuições das ideias de Lev Vygotsky para o ensino de Matemática, por meio da modelagem. Para ele, o contexto histórico-cultural molda o elemento psicológico, determinando a maneira de pensar. Pessoas de diferentes culturas apresentam distintos perfis psicológicos. As funções psicológicas são desenvolvidas ao longo do tempo e mediadas pela interação social, através de símbolos culturais. A linguagem está relacionada à cultura e depende dos fatores sociais. Os conceitos são historicamente construídos e internalizados de maneira particular pelos indivíduos, de forma ampla, integrada, holística e dinâmica. Tais princípios podem contribuir para uma maior compreensão da Modelagem Matemática, uma vez que ela parte de situações apresentadas por problemas reais, alinhados às vivências dos estudantes,*

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levando estes a assumir o protagonismo nesse processo, na medida em que desenvolvem um modelo matemático em um ambiente de aprendizagem no qual a interação se faz presente.

***PALAVRAS-CHAVE**: Processo histórico-cultural. Funções psicológicas. Modelagem matemática.*

***RESUMEN**: El presente estudio es un ensayo teórico que analiza las posibles contribuciones de las ideas de Lev Vygotsky a la enseñanza de las matemáticas, a través de la modelización. Para él, el contexto histórico-cultural configura el elemento psicológico, determinando la forma de pensar. Las personas de diferentes culturas tienen diferentes perfiles psicológicos. Las funciones psicológicas se desarrollan a lo largo del tiempo y están mediadas por la interacción social, a través de símbolos culturales. El lenguaje está relacionado con la cultura y depende de factores sociales. Los conceptos son históricamente construidos e internalizados de manera particular por los individuos, de una manera amplia, integrada, holística y dinámica. Dichos principios pueden contribuir a una mayor comprensión de la modelización matemática, ya que se parte de situaciones que se presentan con problemas reales, alineados con las experiencias de los estudiantes llevándolos a asumir el protagonismo en este proceso, a medida que desarrollan un modelo matemático, en un ambiente de aprendizaje, en el que la interacción está presente.*

***PALABRAS CLAVE**: Proceso histórico-cultural. Funciones psicológicas. Modelización matemática.*

Introduction

The teaching of Mathematics in Brazil has faced many challenges in recent decades, both regarding the teaching and learning processes themselves, as well as the way in which the discipline is viewed by society in general. When a teacher, faced with forty students in a classroom, asks the following question: “Which subject do you like best?”, according to our teaching experience, especially in the final years of Elementary School and in Education Medium, Mathematics is rarely mentioned. In this work, we make some considerations about some central concepts in the work of Russian psychologist Lev Semionovich Vygotsky, to help teachers who teach Mathematics to develop teaching strategies involving modeling.

According to Vygotsky (1996; 2000; 2001), it is collective learning that will promote human development, since man is a social being, the result of an aggregate of social and historical interactions. Man's relationship with the world is not direct but mediated by instruments and signs. The author emphasizes the importance of thought and language (especially referring to speech, speech) in this relationship. He also highlights, in his work, the spontaneous and scientific concepts; the so-called inverse method; the current (actual)

level of development, potential development and near development, or zone of proximal development.

In our analysis, we will adopt as theoretical framework some of Vygotsky's ideas, from the perspective of Marxist Psychology, as well as major references in the Mathematical Modeling field, such as: Bassanezi (1999), Burak (1999), Biembengut (2000), and Barbosa (2004). Considering these theoretical frameworks, we consider Mathematical Modeling as a viable teaching strategy, whether in Basic Education, as provided for in the National Common Curricular Base - BNCC (BRASIL, 2018), or in Higher Education.

We made use of Vygotsky's ideas, in an attempt to change the decontextualized and negative perception of Mathematics manifested by most students. We consider this change essential for the success of educational practices in this discipline, since it is the main subject in the appropriation of knowledge in the teaching and learning processes, considering its historical-cultural context, through its interaction with peers. We also emphasize a basic point: how have mathematical problems been approached? Are they in line with the reality of these students, a key point for meaningful learning, contributing to the formation of autonomous and critical subjects, in the perspective of Skovsmose (2014; 2018)?

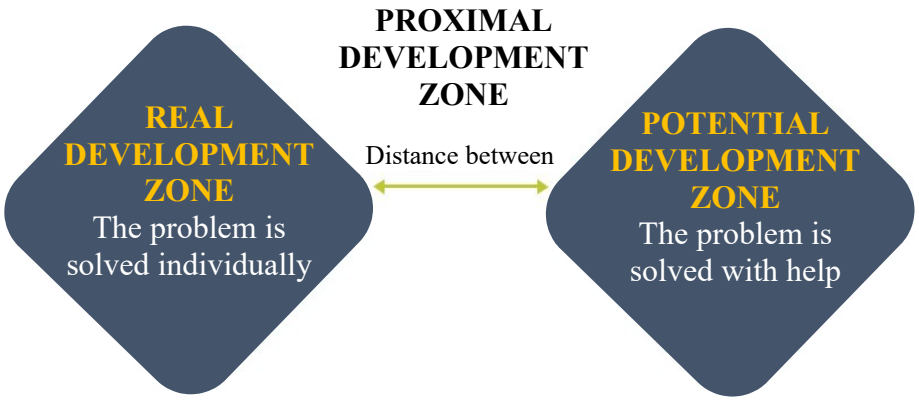
Some conceptions of Vygotsky

We will present, *a priori*, some concepts of this psychologist that has his roots inspired by Karl Marx, more precisely in the historical-dialectical materialism, one of the main methods of sociological analysis regarding the struggle of social classes. Vygotsky (1987; 1996) emphasizes the role of culture in the cognition process, as it is not seen as something static, on the contrary, it goes through a transformation process and will bring as a basic concept about its thinking, what he calls it dialectic. That is, the way in which man acts on and with the world, thus producing culture, and this culture, acting on and with man, transforming him. This author created the concept of mediation, thus described as a social experience that requires participation and collaboration from both students and teachers.

When it comes to development and learning, unlike Piaget's idea, whose focus of the work is on the individual development process as a generator of conditions for learning (LA TAILLE; OLIVEIRA; DANTAS, 2019), Vygotsky assumes that learning generates development and not the other way around. We must emphasize here that this author does not deny biological conditions, as has been widely thought, he just does not rely on a biological perspective to recognize the role of the child in the world.

Vygotsky (2010) emphasizes the so-called superior or specifically human psychological functions, which are mediated by culture. Man is not always the same, man is changing, transforming throughout the entire historical process. To understand the relationship between development and learning, it is necessary to understand what we call the current or real level of development, that is, what the child can accomplish alone, which characterizes mental development retrospectively, the functions already mature; as well as the zone of proximal development, characterized by prospective mental development, abstractions, functions that have not yet matured. Even at this level, the child can perform a task with the help of an adult or in cooperation with others who have already understood the problem, as illustrated in the figure below:

Figure 1 – Relationship between real, proximal and potential development zones



Source: Brasil (n/y)

In relation to mediation, Vygotsky (2000; 2010) highlights two mediating elements that are instruments and signs, which are additional links in this relationship of development and exchange that take place between the individual and the environment in which they are inserted.

In signs, for example, there is the ability to represent the world, it is the mental representations that replace the absent object, for example: language, scientific production, the construction of ideas and concepts. This author breaks with the idea of the stimulus-response relationship. For him, between these two factors there is a mediation process that will act on the child's thinking, leading to a response. Based on this mediation, we must reflect on the educational processes regarding the role of the teacher and the subjects who are present in the child's life, a method called double stimulation.

About thought and language, Vygotsky (1987; 2000) works with the assumption that language is a primordial object of study. It works with two functions of language, which are:

communication, in which people develop the language to communicate, that is, language is born as a form of communication; and generalizing thinking, which is where language fits with thought. It is in this second function that the relationship between thought and language becomes strong. The use of language implies a general understanding of the world. The psychologist also states that the first use of language is what he calls socialized speech, which is the child's speech with others and for others, and also highlights the so-called inner speech, that is, speech for me.

In terms of the study of concept formation, Vygotsky (1987; 1996; 2000) differentiates between spontaneous and scientific concepts. The first is identified as not being aware of the act of thinking, attention is always related to the object, that is, to the concrete. The second concept refers to awareness, however, for the psychologist, this awareness only occurs if the child can explain how to do something.

We cannot think about the development of concepts without considering the context in which we live, as it would be a lifeless way of thinking and without the historical, social and cultural aspects that constitute such processes in the individual and collective relationship of the subject.

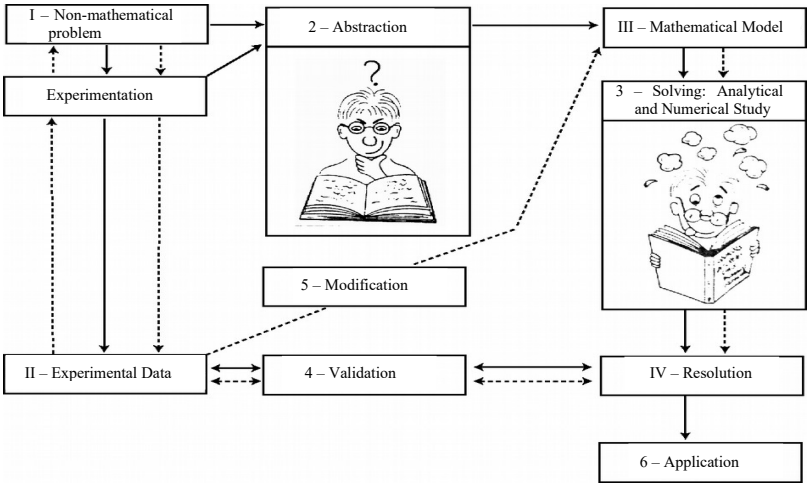
Some conceptions about Mathematical Modeling

In this section, we present the concept of Mathematical Modeling from the perspective of some of the main national references, as it is one of the active methodologies recommended by the BNCC (BRASIL, 2018), a more dynamic alternative, which places students as protagonists in the knowledge appropriation process.

Modeling has been present in our civilization since the beginning, it is considered as old as Mathematics itself. We start our reflection with the concept of Mathematical Modeling anchored in Bassanezi (2011), which uses it to transform reality problems into mathematical problems and solve them by interpreting their solutions in the language of the real world.

The Mathematical Modeling of a situation or problem presented to students presents a sequence of steps that will be visualized in the diagram in the figure below:

Figure 2 – Mathematical Modeling Steps proposed by Bassanezi



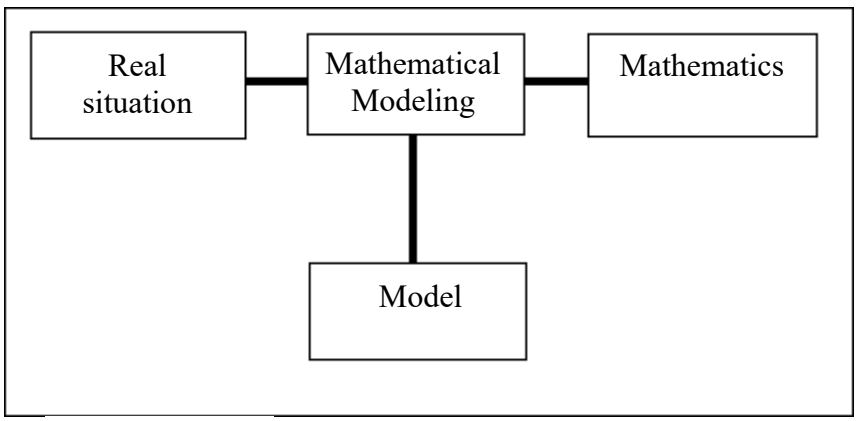
Source: Bassanezi (2011, p. 27)

The steps proposed by Bassanezi (2011) are: experimentation, abstraction, resolution, validation and modification, which will help students in the teaching and learning process, also helping the teaching practice.

According to Bassanezi (1999), Modeling consists of the process of elaborating realistic models defined by the action strategies of each individual on a given reality, impregnated with the interpretations and intersubjectivities peculiar to each modeler.

We will continue with Biembengut (1999). In his view, Mathematical Modeling is a means in which two disjoint sets interact: Mathematics and reality. It is the process that involves obtaining a model, thus awakening in students the interest in learning mathematics content that they did not know before.

Figure 3 – Process for Obtaining a Template



Source: Adapted from Biembengut (1999)

Biembengut (1999) divides these procedures into three stages, called by the author interaction with the subject, mathematization and, finally, the so-called Mathematical Model. These three steps will be subdivided into six sub-steps that we will see next.

In this approach, it is necessary to recognize the problem-situation and become familiar with the subject to be modeled. In this first step, the subject to be studied is defined. In the second stage, the mathematization consists of the sub-steps formulation of the problem (the hypotheses) and the resolution of the exposed problem. It is a stage that will challenge the student. The creativity and experiences that students bring from their lives are of paramount importance in this translation of the real problem into mathematical language. The third and last step, called the Mathematical Model, has as sub-steps the interpretation of the solution and its validation, since it is necessary to check whether the solution found satisfies the conditions of the problem exposed, if not, the process must return to the second step, being reorganized.

Dionísio Burak (2004) understands Mathematical Modeling as an alternative methodology for teaching Mathematics, centered on the interest of those involved in the process. Furthermore, Burak (2004) suggests as a suggestion that modeling activities be carried out considering five different stages: choice of theme; exploratory research; survey of problems; resolution of the problem(s) and the development of Mathematics related to the theme; critical analysis of the solution(s). The author says that:

> Mathematical Modeling is a set of procedures whose objective is to build a parallel to try to explain, mathematically, the phenomena present in the daily life of human beings, helping them to make predictions and make their decisions (BURAK, 1992, p. 62, our translation).

Barbosa (2004) and Barbosa, Caldeira and Araújo (2007) underline that Mathematical Modeling is in the sense of being a learning environment in which students will be invited to investigate and problematize situations of reality through Mathematics. For these authors, an action to be called Mathematical Modeling needs to present two characteristics: the first is to be a realistic problem, of great interest to students, about which students have no previous scheme for resolution, needing to elaborate it. in the process of modeling itself; the second is that such a problem is external to mathematics, that is, the modeling deals with problems that are *a priori* external to this discipline.

One of the points to highlight about modeling is that students will not only solve the proposed problems but will participate in the development of and in the learning processes,

leading the students to a greater interest in the discipline, more than that, also showing the role of Mathematics in society.

Barbosa (2004, p. 04, our translation) adds that: “Students have a little more participation, as they bring the problem and integrate in all stages to solve the problem, that is, they seek information that enables the creation of the model, as well as its validation”.

We agree with the author, as the Mathematical Modeling process is only possible if there is student engagement. It is an authorial work, which contemplates what the BNCC prescribes (BRASIL, 2018), in the sense of promoting student protagonism through active teaching methodologies.

Possible contributions of Vygotskian thought to Mathematical Modeling

In this third section, we seek to articulate the concepts already presented from Vygotskian thought with Mathematical Modeling, as an alternative proposal both for teacher planning and for classroom practices.

The teacher who teaches Mathematics needs to consider the assumption that the student is the result of the historical-cultural context in relation to the environment in which they live. Thus, we cannot understand that all students learn in the same way and at the same pace (BRASIL, 2018). Culture and socialization play a crucial role in its development because, for Vygotsky, there is only learning from the other. In the absence of the other, man does not build himself as a man. From this perspective, formation takes place in the relationship between the subject and the society around him. In this way, the individual changes the environment and the environment change it back.

In our teaching experience, we observe that, when we teach Mathematics classes through Mathematical Modeling in Kindergarten and early years of Elementary School, children present spontaneous concepts, in which they are not fully aware of their thoughts, their gaze is object-oriented, hence the importance of working with concrete, in the case of counting and mathematical operations.

For elementary school students (final years), high school and college, we can emphasize scientific concepts, as they already have the maturity and level of knowledge necessary for a deeper understanding of the mathematical concepts that will be worked on in modeling activities, for example, the concept of affine function. Hence the importance of the teacher carrying out activities in which the student not only presents the answer, but also manages, in a conscious and critical way, to justify it.

When a teacher makes use of Mathematical Modeling, whatever the level of education, they are helping students to realize that the Mathematics discipline can make sense in their lives, making them more critical and instigating them to search for scientific knowledge.

Vygostky (1987; 2000; 2010), regarding thought and language, can offer important contributions to Mathematical Modeling, in the sense that when the student is solving a problem, preferably collaboratively, in small groups, communication between peers is essential. The use of mother tongue and mathematical language is present in their social interactions in the didactic activity, as Machado (1994) argues, making these students develop skills (BRASIL, 2018) that give them a secure mastery of the mathematical content about the which is interacting with colleagues and with the teacher himself.

The mathematical learning process, through modeling, must be mediated, either by the teacher or even by classmates who have already matured or learned such mathematical concepts. In short, from mathematical concepts that have already matured (real development zone) we can, in/through mediation, contribute to potentialize the next level of development, that is, those that have not yet matured.

Vygotsky (1987; 1996), influenced by Marxist thought, especially regarding historical and dialectical materialism, defines it as a method of interpreting reality, an intrinsically dynamic and contradictory reality, dialectically feeding on new discoveries and new questions. Through these new questions, new knowledge is built individually and collectively.

This method implies that our objective reality is historical, diachronic, followed attentively and participatively throughout history. This allows us to analyze our reality as an evolving process. Therefore, it is necessary to understand our experiences in a society that alienates and is alienated to deconstruct it with and through critical thinking.

Students need to be aware of the society they live in and the capitalist model in which they are inserted and that over time they will no longer be the same, they will be in a process of constant change, considering their constitution as a unconcluded subject, unfinished. In this context, we ask: what is the contribution of these ideas to Mathematical Modeling?

It is necessary that the professors themselves take ownership of these studies and dare, armed and constituted in/by and with the resources of the active methodologies proposed in the BNCC (BRASIL, 2018). D'Ambrosio (2012, p. 95, our translation) asserts that:

> When starting the class, the teacher has great freedom of action. Saying you cannot do this or that is an excuse. It is often difficult to do what is intended but falling into a routine is exhausting for the teacher. [...] No professional

> should do the same thing for more than four or five years. The apparent acquisition of an execution routine leads to lack of creativity and consequently to inefficiency, but what is more serious is stress [...]. The International Labor Organization indicates that teaching is one of the most stressful professions.

Realistic mathematical problems, contextualized in the universe of students' interests, aligned with the needs of the local community, with the help of modeling, allow for the development of *materacia*[4] (D'AMBROSIO, 2012; SKOVSMOSE, 2014).

The main objective of the teaching and learning processes is to provide conditions for the development of citizens who know their history, who reflect critically (SKOVSMOSE, 2014; 2018) and are capable to fight for the construction of a better society.

Final considerations

We seek to reflect on how Vygotsky's ideas can greatly contribute to help teachers' practices in the field of Mathematical Modeling, a psychologist influenced by Marx's ideas, in relation to historical and dialectical materialism, highlighting his importance for critical mathematics education. We present students with a lively, dynamic Mathematics, centered on their reality, in a way that is meaningful to them, that is, that in and through their relationship with each other they can produce meanings at school and in other social spaces.

We intend, therefore, to contribute to the development of autonomous students, capable of thinking, reflecting, knowing their history and the context in which they live and discussing about and with it, capable of fighting for equity in the capitalist society to which they are inserted and that still glimpse social equality.

Faculty and students need to work collaboratively. We hope that this work will help teachers who teach Mathematics, while planning at all levels of education for a citizenship formation. When a teacher knows the reality of their student, the teaching and learning processes flow more constructively, as the student feels free to share their doubts, emotions, achievements, anxieties and even their fears, which is extremely important in the current context experienced in the world by the COVID-19 pandemic.

The educator and the student need to recognize and assume their roles in society, understanding the relevance of teaching and learning mathematics in this empowerment. We believe, therefore, to contribute to a better acceptance of Mathematics in their lives,

[4] D'Ambrosio understands *materacia* as the ability to interpret and handle signals and codes and to propose and use models in everyday life.

punctually, by understanding its importance at school and in other historical and social contexts.

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