FORCE CONCEPT: A VECTOR GREATNESS MOBILIZED BY MECHANICS AND THE BASE OF ENGINEERING COURSES

CONCEITO FORÇA: UMA GRANDEZA VETORIAL MOBILIZADA PELA MECÂNICA E BASE DOS CURSOS DE ENGENHARIA

CONCEPTO DE FUERZA: UNA GRANDEZA VECTORIAL MOVILIZADA POR LA MECÁNICA Y BASE DE LOS CURSOS DE INGENIERÍA

Viviane RONCAGLIO[1]
e-mail: roncaglioviviane@gmail.com

Cátia Maria NEHRING[2]
e-mail: catia@unijui.edu.br

Isabel Koltermann BATTISTI[3]
e-mail: isabel.battisti@unijui.edu.br

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[1] Regional University of the Northwest of the State of Rio Grande do Sul (UNIJUÍ), Ijuí – RS – Brazil. Member of GEEM. Doctorate in Science Education.
[2] Regional University of the Northwest of the State of Rio Grande do Sul (UNIJUÍ), Ijuí – RS – Brazil. Teacher. Leader of the GEEM. Doctorate in Education (UFSC).
[3] Regional University of the Northwest of the State of Rio Grande do Sul (UNIJUÍ), Ijuí – RS – Brazil. Professor. Member of GEEM. Doctorate in Science Education (UNIJUÍ).

ABSTRACT: This writing aims, from a bibliographic review, to constitute a theoretical contribution that makes possible the understanding of the Force concept as a vector quantity, from the field and the conceptual relationships that constitute it. In order to do so, we considered as material for analysis the force definitions presented in book chapters of the basic bibliography of the Physics I and General Mechanics I disciplines of a Civil Engineering Course. The theoretical foundation that supports this writing is the Historical-Cultural Theory. The methodological approach used is the Discursive Textual Analysis of Moraes and Galiazzi (2016), from which we constitute the units of meaning, categories and propositions. From the analyzes carried out, we concluded that the approach given to the force concept is strongly related to a vector quantity and the intuitive notions of pushing or pulling.

KEYWORDS: Historical-cultural theory. Conceptual appropriation. Learning. Mathematical concept. Civil Engineering.

RESUMO: *Esse estudo tem por objetivo, a partir de uma revisão bibliográfica, constituir um aporte teórico que possibilita a compreensão do conceito Força como uma grandeza vetorial, a partir do campo e das relações conceituais que o constituem. Para tanto, consideramos como material de análise as definições de força apresentadas em capítulos de livros da bibliografia básica das disciplinas de Física I e Mecânica Geral I de um Curso de Engenharia Civil. A fundamentação teórica que sustenta a escrita é a Teoria Histórico-Cultural. O percurso metodológico utilizado é a Análise Textual Discursiva de Moraes e Galiazzi (2016), a partir da qual constituímos as unidades de significado, as categorias e as proposições. A partir das análises realizadas, concluímos que a abordagem dada ao conceito força está fortemente relacionada a uma grandeza vetorial e às noções intuitivas de empurrar ou puxar.*

PALAVRAS-CHAVE: *Teoria histórico-cultural. Apropriação conceitual. Aprendizagem. Conceito matemático. Engenharia Civil.*

RESUMEN: *Este escrito tiene por objetivo, a partir de una revisión bibliográfica, constituir un aporte teórico que posibilite la comprensión del concepto Fuerza como una de las magnitudes vectoriales, desde el campo y las relaciones conceptuales que lo constituyen. Para ello, consideramos como material de análisis las definiciones de fuerza presentadas en capítulos de libros de la bibliografía básica de las asignaturas de Física I y Mecánica General I de una carrera de Ingeniería Civil. La fundamentación teórica que sustenta la escrita es la Teoría Histórico-Cultural. El enfoque metodológico utilizado es el Análisis Textual Discursivo de Moraes y Galiazzi (2016), a partir del cual constituimos las unidades de sentido, categorías y proposiciones. A partir de los análisis realizados concluimos que el abordaje aplicado al concepto de fuerza está fuertemente relacionado con una magnitud vectorial y las nociones intuitivas de empujar o tirar.*

PALABRAS CLAVE: *Teoría histórico-cultural. Apropiación conceptual. Aprendizaje. Concepto matemático. Ingeniería Civil.*

Introduction

Society is in constant evolution, which enables the emergence of new technologies and means of communication that allow access to all and any kind of information. This requires skills of analysis and synthesis, in order to allow each individual to understand and intervene in this increasingly complex and, consequently, increasingly competitive context. The work understood as a human activity (RONCAGLIO; BATTISTI; NEHRING, 2021) requires trained professionals with diverse skills, with theoretical and practical knowledge essential to meet the demands of contemporary society and, therefore, a qualified professional training that meets these expectations and needs is required. From this perspective, higher education is a formal process of teaching responsible for professional training, capable of allowing the development of competencies and skills that enable the subject to act efficiently and consciously in different realities.

In the professional education processes, the Civil Engineering course, which is the focus of this study, needs to promote in the students the development of specific competences and abilities, focused on their area of work, such as supervision, coordination, and technical guidance; technical and economic feasibility study; assistance, consulting, and advising; direction of works and technical services; survey, expertise, arbitration, reports, and technical opinions; budgeting; execution and completion of works and technical services; performance of technical positions and functions; technical work and team leadership in installation, assembly, operation, repair, or maintenance; execution of technical drawings (RONCAGLIO; BATTISTI; NEHRING, 2021). These competences require theoretical and practical knowledge that needs to be learned by future engineers during their professional training course.

The relationship of the subject with the reality that his profession encompasses is not a direct relationship, it is mediated by different concepts apprehended from processes that involve signification. The appropriation of such concepts strongly enhances the conditions of the subject's professional constitution. In the case of the engineering profession, we highlight two fundamental sciences, Mathematics and Physics, and the concepts discussed by these sciences are the basis for the study of Engineering. In Mathematics, the concept Vector is treated with approaches from the fields of arithmetic, geometry and algebra. In Physics, the concept Force is a vectorial quantity that needs a module, a direction and a sense to be represented, that is, it needs the concept Vector for its representation. These two sciences correlate with each other, Physics uses Mathematics as a tool to explain and/or represent physical phenomena, as is the case of the concept of Force and the concept Vector.

In this sense, considering the attributions of a higher education course, as well as the importance of the apprehension of scientific concepts, as mediating elements, by the students, the concepts that integrate the curricular program of the disciplines need to be understood with and from the field and the hierarchical relations of which they are part. Given the above, the present writing, which is part of a larger research, aims to: constitute a theoretical contribution that enables the understanding of the concept of force, as a vector quantity, from the field and the conceptual relations that constitute it. This objective is delimited from the following question: What is the approach given to the concept of Force in the books of the basic bibliography of the disciplines Physics I and General Mechanics I, which introduce and discuss this concept in the Engineering Courses?

The appropriation of concepts in the historical-cultural perspective

Based on the historical-cultural approach, we understand knowledge as a result of human activity in experiences with the physical and social environment, and that human learning has, therefore, a social nature. In this perspective, the social aspect and the appropriation of scientific concepts by the subjects are extremely important, since human development depends on established interactions mediated by signs and instruments.

When the subject appropriates the meaning of a scientific concept, the relationship established with it is mediated by other concepts. In the historical-cultural perspective, a concept is not formed alone, but in relation to others, inserted in a conceptual system. For Vygotsky (2008, p. 104, our translation):

> [...] a concept is more than the sum of certain associative connections formed by memory, it is more than a mere mental habit; it is a real and complex act of thought that cannot be taught by training, but can only be realized when the child's own mental development has already reached the necessary level (our translation).

From this perspective, the meaning of a concept is closely related to abstraction and generalization processes and corresponds to the evolution of levels of apprehension and meaning of words. The appropriation of concepts happens through abstractions and generalizations, which are related to the mobilization of different types of representation of a concept expressed by a system of signs. For Vigotski (2008, p. 104, our translation), a concept expressed by a word represents an act of generalization.

> When a new word is grasped by the child, its development has barely begun: the word is first a generalization of the most primitive kind; as the child's intellect develops, it is replaced by generalizations of an increasingly higher kind - a process which eventually leads to the formation of true concepts. The development of concepts, or of the meanings of words, presupposes the development of many intellectual functions: deliberate attention, logical memory, abstraction, the ability to compare and differentiate.

Thus, the appropriation of the meaning of a concept by the academic involves the development of superior mental functions and these are related to the structure of the concept, its formation, the meaning produced in different contexts and the establishment of conceptual relations. The appropriation by the Engineering student of the concept of Vector expands the conditions for attributing new meanings to his world, that is, to the context of the profession, broadens his horizons of perceptions and makes it possible to modify the form of interaction with the reality that surrounds him. This internalization or appropriation of the concept by the student occurs from the outside in, from the social to the individual, being a complex process that requires a deliberate and intentional interaction. It is not enough to present the concept to the student, he needs to understand the historical and cultural context, it means appropriating the social experiences built historically by humanity. Once internalized, this concept is part of the acquisitions of the subjects' development, and can become a tool of their thinking. In Historical-Cultural Theory, the appropriation of concepts involves "[...] the internal reconstruction of an external operation" (VIGOTSKY, 1994, p. 74, our translation), which consists of a series of transformations, which are:

> a) An operation that initially represents an external activity is reconstructed and begins to occur internally. Of particular importance for the development of higher mental processes is the transformation of the activity that uses signs, whose history and characteristics are illustrated by the development of practical intelligence, voluntary attention, and memory.
> b) An interpersonal process is transformed into an intrapersonal process. All functions in the child's development appear twice: first on the social level, and then on the individual level; first between people (interpsychological), and then within the child (intrapsychological). This applies equally to voluntary attention, logical memory, and the formation of concepts. All higher functions originate in real relations between human individuals.
>
> c) The transformation of an interpersonal process into an intrapersonal process is the result of a long series of events occurring throughout development. The process, being transformed, continues to exist and change as an external form of activity for a long period of time before it becomes definitively internalized. (VIGOTSKY, 1994, p. 75, our translation)

During this process, learning occurs, which enables transformations in the higher psychic functions and, consequently, the development of the subject. Learning here is considered fundamental to the process of development of the higher psychological functions; for Vygotsky (1994, p. 108, our translation), learning generates development. "Learning is more than the acquisition of the ability to think; it is the acquisition of many specialized abilities to think about various things." In this respect:

> [...] learning is not development; however, properly organized learning results in mental development and sets in motion various developmental processes that would otherwise be impossible to occur. Thus, learning is a process of development of culturally organized and specifically human psychological functions (VIGOTSKY, 1994, p. 118, our translation).

For Vigotsky (1994), learning is a necessary aspect for the development of superior psychological functions, which are organized by culture and are characterized as specifically human. In this way, human development is intrinsically related to the sociocultural interaction of man, that is, the formation of the human psyche, of mental, affective, psychomotor capacities, before being developed by man, are already available in society, through culture. Therefore, development is prevented from happening in the absence of situations conducive to learning – it requires intentional pedagogical action, learning occurs from interactions with the other. However, it is worth noting that learning results in the development of the subject when he mobilizes superior mental operations, based on analysis and synthesis processes, in different contexts.

Methodological procedures

The research presented here is based on the methodological framework of Textual Discourse Analysis - TDA - by Moraes and Galiazzi (2016). The TDA is structured in three stages: unitarization, categorization and metatext. The first stage of TDA, unitarization, is the initial movement of the analysis, which requires a careful and deep reading of the data. It is marked by disorder, the moment of data deconstruction, in which the researcher, while analyzing the data, performs several interpretations. From this movement emerge the units of meaning.

We considered, as material for analysis, the definitions presented in chapters of the books of the basic bibliography proposed in the Teaching Plan of the subjects Physics I and General Mechanics I of the Civil Engineering Course of the institution where the first author developed her doctoral studies and where the other two researchers work as teachers. Table 1 presents the referred bibliographies.

We chose to analyze chapters of the books, as we consider that this is an instrument that serves as a guide for the teacher and as a resource for student learning. The books listed in the basic bibliography are intended to guide the teacher in what will be proposed, its emphasis, and approach in the discipline, and may present elements that support the organization of teaching. As for the students, it certainly broadens the study conditions, involving the structuring concepts and procedures necessary for their education.

We considered the subjects Physics I and General Mechanics I because they initiate discussions about Mechanics and about the concept of Force in the Civil Engineering Course. It is worth mentioning that these subjects are part of the curricular program of all the Engineering courses at the University considered in this study, and are called common core subjects, or basic core subjects of the referred courses.

In this production, we are considering the definitions and concepts presented in the basic bibliography books of both disciplines, mentioned above, related to the concept of Force. In this perspective, we understand that it is possible to elaborate a theoretical contribution that enables the understanding of this concept as a vectorial quantity, from the field and the conceptual relations that constitute it.

Chart 1 – Reference of the books of Physics I and General Mechanics I, analyzed in this production

Subjects	Bibliography
Physics I	**Livro F1:** HALLIDAY, D.; RESNICK, R. **Fundamentos de Física**: Mecânica. 9. ed. Rio de Janeiro: LTC, 2012. v. 1.
	Livro F2: NUSSENZVEIG, H. M. **Curso de Física Básica 1:** Mecânica. 3. ed. São Paulo: Edgard Blucher, 1981.
	Livro F3: YOUNG, H. D.; FREEDMAN, R. A. **Sears e Zemansky** – Física 1: Mecânica. 12. ed. São Paulo: Pearson Addison Wesley, 2008.
General Mechanics I	**Livro M1:** BEER, F. P. et al. **Mecânica vetorial para engenheiros**: estática. Porto Alegre: AMGH, 2012.
	Livro M2: HIBBELER, R. C. **Estática**: Mecânica para Engenharia. São Paulo: Pearson Prentice Hall, 2015.
	Livro M3: MERIAM, J. L.; KRAIGE, L. G. **Mecânica para Engenharia**: estática. Rio de Janeiro: LTC, 2013. v. 1.

Source: Research production

From the analysis performed in this first movement, in the unitarization of data, we defined the following meaning units, Mechanics - fundamental principles and concepts of Mechanics and Force - fundamental concept of mechanics. Once the meaning units were defined, we reached the second stage of TDA, the categorization, where a constructive movement is performed.

> Categorization is the moment of synthesis and organization of a set of information related to the investigated phenomena. These syntheses are the theorizations of the researcher, produced from implicit theoretical perspectives of the research subjects and the researcher himself, always in dialogue with other theorists. They require continuous improvement, adequacy, and refinement during the process of analysis and written production. The categorization process is a research movement strategy that goes from the empirical to the abstract, from the data collected to the theories constructed or reconstructed by the researcher (MORAES; GALIAZZI, 2016, p. 112-113, our translation).

That is, considering this methodological reference, the categories of analysis that gave body to this study are produced. Table 2 presents the unit of meaning and the categories considering the referential of the TDA, and to better examine the intentionality of the research, it is also presented the propositions (third stage of the TDA), defined from the analyzed corpus. The proposition is structured from the capture of the emergent in which the new understanding is communicated and validated, considering the empirical/theoretical relationship. It is the construction of a metatext by the researcher making considerations in relation to the categories of analysis he has built and the units of meaning identified in the data produced. It is a writing that seeks to present in a clear and objective way the researcher's understanding of the data analysis related to the rationale that supports the study.

Chart 2 – Meaning units, categories and propositions of the corpus

Meaning units	**Categories**	**Propositions**
Mechanics - fundamental principles and concepts of Mechanics.	Mechanics as a branch of physics.	Mechanics - a branch of Physics that mobilizes the vector quantity force.
Force - a fundamental concept in Mechanics.	Conceptual Approach to Force in books from the basic bibliography of Physics I.	The concept force integrates a conceptual system. Vector, a mathematical tool for the representation of force.

Source: Research production

Mechanics: Fundamental principles and concepts

Mechanics as a branch of Physics

In this category, we seek to bring the understanding of Mechanics from the approach given in the books of the basic bibliography of the discipline General Mechanics I, the first specific discipline of Engineering Courses that discusses Force and consequently mobilizes Vector. Moreover, it discusses fundamental concepts related to Engineering that are covered in other specific disciplines of the course. To do so, we present discussions based on the following proposition: "Mechanics - a branch of Physics that mobilizes the vectorial quantity Force".

Mechanics: a branch of Physics that mobilizes the vectorial quantity force

Mechanics can be defined as a branch of physics that describes and predicts the conditions of rest or motion of bodies under the action of forces. It can be understood from the mechanics of rigid bodies, the mechanics of deformable bodies, and the mechanics of fluids. The mechanics of rigid bodies is subdivided into two parts, statics and dynamics. Static mechanics studies bodies at rest, and is an essential part of mechanics, widely used by engineers, since it is applied in situations involving, for example, the calculation of how much weight a crane can lift, or the calculation of the force on a point on a bridge and whether its structure can support such force, or to determine the force exerted by the water that a dam in a river needs to support, or how much force a locomotive needs to stop a freight train, among other applications. Dynamics, on the other hand, studies bodies in motion, such as designing a building to resist earthquakes, or calculating how much force is needed to accelerate a 300,000-ton cruise ship or the force to put a satellite in orbit. In rigid body mechanics, the bodies considered are perfectly rigid, i.e., they do not undergo any kind of deformation (BEER *et al*., 2012).

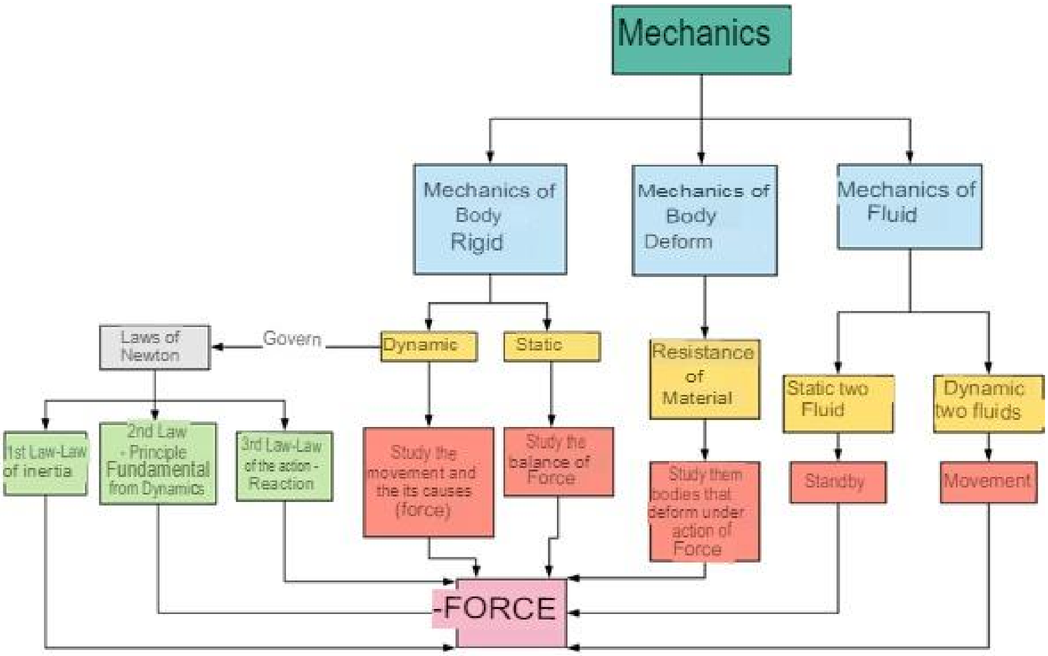
The mechanics of deformable bodies studies bodies that deform under the action of forces. It is the part of mechanics that analyzes structures and materials, considering resistance under forces or loads that can cause deformation. Fluid mechanics is subdivided into fluid statics, which studies the behavior of fluids in motion and at constant speed, i.e., at rest, and fluid dynamics, which studies the behavior of fluids in motion. This part of mechanics studies situations such as the equilibrium of floating bodies - ships, or the wind's action on civil constructions, or the calculation of hydraulic installations - pumping installations in buildings, or steam installations - boilers, or even the action of fluids on submerged surfaces - dams, among several other situations. In Civil Engineering, the study of transport phenomena and hydraulics is based on.

According to Beer e*t al*. (2012), the purpose of mechanics is to explain and predict physical phenomena, establishing the foundations or tools for applications in Engineering. The basic concepts used in mechanics are space, time, mass, and force, among which Force (the focus of this study), a vectorial quantity, is generally defined in the analyzed bibliographies as an action of one body over another. It can be exerted by direct contact or at a distance, and is characterized by its intensity, the point of application, and its direction, represented by a vector. Force is a fundamental physical concept in the study of mechanics, an essential branch in Engineering Courses. Meriam and Kraige (2013, p. VII, our translation) contribute to the discussion by pointing out that the goal:

> [...] fundamental of the study of Mechanics in Engineering is to develop the ability to predict effects of forces and motions when performing creative engineering design functions. This ability requires more than just knowledge of the physical and mathematical principles of mechanics; it also requires the ability to visualize physical configurations in terms of real materials, real constraints, and practical limitations that guide the behavior of machines and structures.

That is, the study of mechanics involves a range of concepts that are necessary for its understanding, a network of concepts articulated from relationships, which define and, when considered, expand the conditions of appropriation of such knowledge by the subjects that investigate it, as indicated in Figure 1.

Figure 1 – Map of Relationships in the Study of Mechanics in Engineering Courses



Source: Prepared by the authors

The map of constituent relations of a conceptual network presented in Figure 1 marks an important point addressed by the theoretical foundation of this study, which argues that a concept is formed from the relationship with others, being inserted into a conceptual system, because a concept is never constituted alone, in an isolated or fragmented way. In other words, the map presents, in the form of a synthesis, the relationships observed in the study of mechanics, from the perspective of the Engineering Course. It is worth mentioning that the construction of the map was based on the analysis of the General Mechanics I textbooks.

The map of relationships in the study of mechanics in Engineering courses makes clear the importance of understanding and appropriation of the vectorial quantity Force to work with mechanical definitions. The understanding of this quantity is fundamental because mechanics essentially explores situations involving force. In this sense, the next category of analysis seeks to present this vectorial magnitude, which is a physical concept, from Physics, which is responsible for introducing basic and essential concepts in Engineering Courses.

Force: fundamental concept of mechanics

Conceptual approach to force in books from the basic bibliography of Physics I

This category presents definitions related to the concept of Force addressed in books of the basic bibliography of the subject of Physics I. We understand that the way this concept is presented in some chapters in the books of this subject interferes significantly in the process of conceptual apprehension by the students, remembering that this subject is responsible for the introduction of basic concepts that are explored in the study of mechanics and consequently in several specific disciplines of Engineering Courses, especially in Civil Engineering Courses. To this end, we present below the discussion based on two propositions: the first, the concept of force integrates a conceptual system, and the second, vector, a mathematical tool for the representation of force.

The concept force integrates a conceptual system

Considering the analyses performed in order to identify the definitions presented for the concept of force, we found in book F1 the initial mention of force presented as follows:

> We shall now define the unit of force. We know that a force can cause a body to accelerate. So, we define the unit of force in terms of the acceleration that a force gives to a reference body [...]. This body has been assigned, exactly and by definition, a mass of 1 kg. We place the standard body on a frictionless horizontal table and pull it to the right until, by trial and error, it acquires an acceleration of 1 m/s2. We then state, by way of definition, that the force we are exerting on the standard body has a modulus of 1 newton (1N). A force is therefore measured by the acceleration it produces. However, acceleration is a vector quantity because it has a modulus and an orientation. Is force also a vector quantity? We can easily assign an orientation to the force (it is enough to assign the orientation of the acceleration), but this is not enough. We must prove experimentally that forces are vector quantities. In fact, this was done a long time ago. Forces are indeed vector quantities: they have a modulus and an orientation and they combine according to the vector rules (HALLIDAY; RESNICK, 2012, p. 92, our translation).

In the book F1, the concept Force is related to the acceleration of a body, which occurs, in the specific case presented by the book, by a "push" and the idea of vector magnitude. Besides the intuitive notion of Force, the book presents a conceptual approach by defining this concept as a vectorial quantity and by highlighting that the work with these quantities follows the vectorial rules, i.e., from a mathematical approach. This fact marks our understanding regarding the need for the concept Vector as a mathematical object to be considered when dealing with the vectorial magnitude Force.

The book F2 presents the concept from a relation with the intuitive idea of muscular effort, that is, a way to define Force, considering that it is a universal concept and that it can have countless interpretations.

> [...] We all know from experience that motion is affected by action that we usually call "forces. Our intuitive idea of forces is related to muscular effort, and we know that by exerting "forces" of this kind we are able to set objects in motion or, more generally, to change their state of motion. [...] A force produces different effects depending on the direction and direction in which it is applied, which suggests a representation of the vectorial type (NUSSENZVEIG, 1981, p. 89-90, our translation).

As we can see, the book F2 establishes a relation with intuitive notions of the concept, besides the approach at a conceptual level when it brings the constitutive elements of the vector magnitude (direction and sense), highlighting the need of the concept Vector for the representation of Force, "a representation of the vector type". F3 relates the concept Force to the idea of muscular effort, pulling or pushing. Moreover, he points out that such concept is a vectorial quantity, after all we can push or pull an object in different positions, which marks a conceptual approach.

> In everyday language, to exert a force means to pull or to push. A better definition is that a force is an interaction between two bodies or between a body and its environment. So, we always refer to the force that one body exerts on another. When you push a car stuck in the snow, you exert a force on it; a steel cable exerts a force on the beam it supports in a building; and so on. [...] force is a vector quantity; you can push or pull a body in different directions (YOUNG; FREEDMAN, 2008, p. 106, our translation).

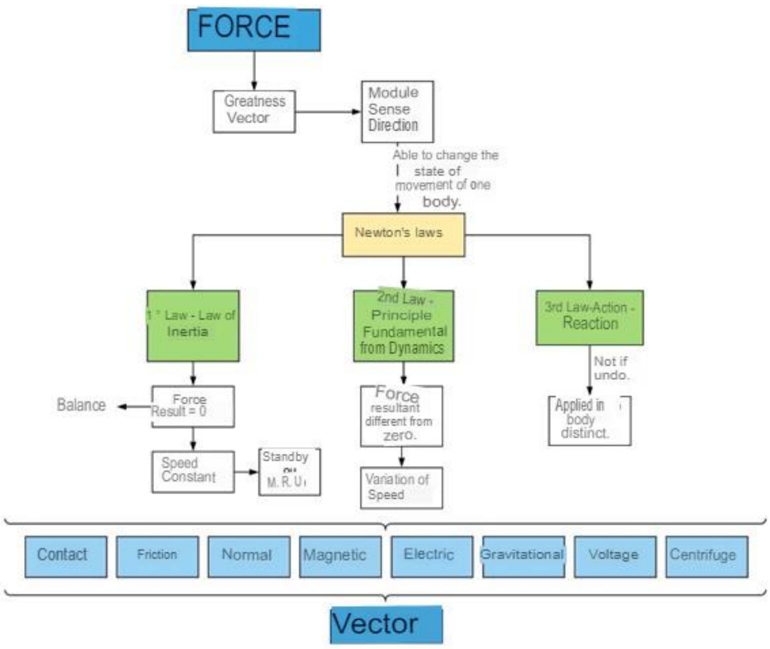
Both concepts presented in the books do not present a formal definition for the physical concept: the conceptual notion of what Force means does not appear in the analyzed books. What the books present are constituent ideas of the concept force, such as the idea of vectorial magnitude, the need for sense, direction and intensity, elements that constitute the conceptual system.

The relationships presented start from an intuitive notion of an effort that generates a movement of a body or object, such as pushing and pulling. Force is a scientific concept, in the area of Physics, it is an action that causes motion - acceleration, displacement of objects, or deformation. In addition, there are several types of force, such as the normal force, frictional force, tension force, centripetal force, inertial force, among others that Physics considers.

The three books, when referring to the concept Force, relate it to a vectorial quantity, that is, Force is defined according to its module (represents its intensity), direction (directions in which forces apply) and direction (positive, negative, up and down, right and left...). They cite Newton's laws as the laws that define the behavior of forces: the first law - Law of Inertia (if a body is at rest it tends to remain at rest, if it is in motion it tends to remain in motion with constant velocity), the second law - Fundamental Principle of Dynamics (the resultant force acting on a body is equal to the product of the acceleration of a body by its mass) and the third law - Law of Action and Reaction (the action and reaction forces always have the same module and act in the same direction, but have opposite directions).

Newton's Laws describe the causes that can change the state of motion of a body and are configured as the basis for understanding Mechanics, a branch of Physics responsible for the study of movements, as we saw in the item. Moreover, we highlight here again Force as the agent responsible for changing the state of motion of a body, altering its speed. In other words, Force is a central concept in Physics, it is from it that many physical phenomena can be analyzed and improved, and consequently it has significantly contributed for mankind to evolve. It is a basic concept for the study of most of Physics and, like other concepts, it is also part of a conceptual system, as shown in Figure 2 below.

Figure 2 – Map of relationships in the study of force in Physics I



Source: Prepared by the authors

The concept Force allowed the appearance of Newton's Laws and is also the basis for the study of Mechanics, it is a vectorial quantity, and to be represented, it needs the mathematical concept Vector: it is only possible to represent force using Vector. The concepts discussed in the books analyzed here and that were the basis for the construction of the concept map, when presenting such concept, make this relationship, highlight Force as a vectorial quantity and suggest a representation of the vector type, i.e., from vectors.

In general, Force is a physical concept, a vectorial quantity, fundamental in Physics and the basis of the study of Mechanics. It is used in mechanics to explore situations and/or analyze phenomena, as well as to define variables and solve problems. However, this is only possible through the geometric or algebraic representation of the concept Force. That is, through the vector, considering the geometric and/or algebraic representation of situations involving force, it is possible to explore, analyze, define variables, find the problem and/or point solutions. Thus, Vector is an essential concept in this process, it is the concept that allows the representation of situations involving Force.

So, we say that Force is a physical concept based in mechanics and that needs the Vector, a mathematical concept that allows, through its geometric representation, that this process of geometric representation of Force occurs. To better understand this mathematical concept and its importance in the engineer's professional education process, we present below an item that deals with the Vector concept as a mathematical tool for the representation of Force.

Vector, a Mathematical Tool for the Representation of Force

Vector is a mathematical concept that, according to the study already developed (RONCAGLIO; BATTISTI; NEHRING, 2021), is initially discussed in the subject of Analytic Geometry and Vectors or in the subject of Linear Algebra, which are the disciplines responsible for introducing this concept in higher education, especially in courses of the Exact Sciences, such as Mathematics and Engineering. This concept is explored in the first semesters of the courses, usually in the second or third semester. Roncaglio, Battisti and Nehring (2021), corroborate the discussion by presenting a table with disciplines of Engineering Courses (Civil, Electrical and Mechanical) that introduce the concept Vector. The table presented by the authors reinforces the idea of a common core of disciplines that discuss and introduce the concept Vector, and mark this concept as basic in professional training.

In this sense, the research (RONCAGLIO; BATTISTI; NEHRING, 2021) reinforces the idea of Vector as a fundamental basic concept for the professional education of the Engineer, since it highlights the specific disciplines of the Civil Engineering Course that mobilizes Vector. In addition, some of the disciplines mentioned in this article are part of the common core of the courses and, therefore, point to the mobilization of this concept in all Engineering courses. Thus, Vector is a basic mathematical concept in Engineering Courses and essential to the professional constitution of Engineers.

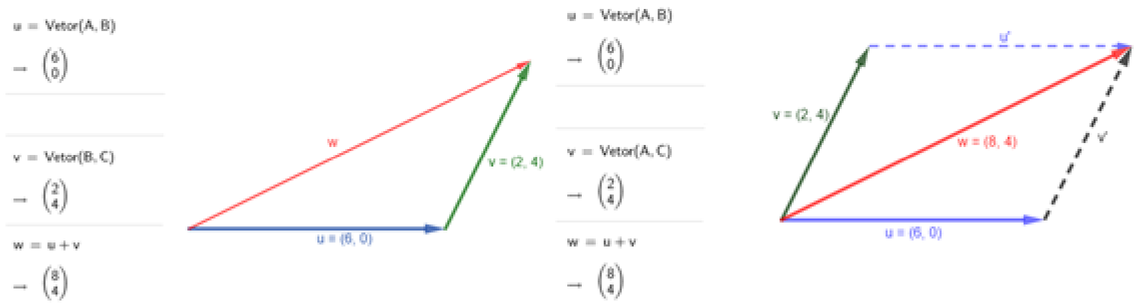
Mathematically, Vector is defined by Santos (2007, p. 2-3, our translation) as:

> **Definition 1 -** We will say that the oriented segment AB is equipollent to the oriented segment A'B' if one of the following three statements is true:
> 1. A = B and A' = B'.
> 2. AB and A'B' are situated on the same line and it is possible to slide A'B' on this line so that A' coincides with A and B' coincides with B.
> 3. The figure obtained by connecting the points A to B, B to B', B' to A' and A' to A is a parallelogram.
> Note that two points (when considered as oriented segments) are always equipollent. The reader can easily show that the equipollence relation satisfies the following properties:
> I - Reflexivity: every oriented segment in space is equipollent to itself.
> II - Symmetry: if the oriented segment AB is equipollent to the oriented segment A'B', then A'B' is equipollent to AB.
> III - Transitivity: if the oriented segment AB is equipollent to the oriented segment A'B' and if A'B' is equipollent to the oriented segment A'B', then AB is equipollent to A'B'.
> Because of the three properties mentioned, it is usual to say that equipollence is an equivalence relation.
> **Definition 2 -** The vector determined by a prime segment AB is the set of all prime segments in space that are equipollent to the prime segment AB.
> The vector determined by AB will be denoted by AB; the oriented segment AB is a representative of the vector AB. It is convenient to represent both the oriented segment AB and the vector AB by an arrow with origin at A and end at B. The reader should, however, keep in mind that this is an abuse of notation: the oriented segment AB and the vector AB are distinct mathematical objects, since AB is an oriented segment (i.e., a set of points), while AB is a set of oriented segments.
> Note that the oriented segments AB and CD represent the same vector if, and only if, these segments are equipollent. Therefore, the same vector can be represented by an infinite number of different segments. In fact, if AB is a prime segment and P is any point in space, the reader can easily see that there is one, and only one, prime segment PQ, with origin at P, such that PQ is equipollent to AB. It follows that the vector AB has exactly one representative at each point in space.

Equipole relationship, equivalence relationship, oriented segments, are concepts that structure the concept Vector and support the operations involving this concept. For example, when we perform the addition of two vectors in their geometric representation, we can do so considering two rules, the polygon law - used when the origin of one vector is the end of the other, the vector summed in this case is the one that closes the triangle, as shown on the left in

Figure 3.

Figure 3 – Polygon law (right) and parallelogram law (left))



Source: Prepared by the authors

And the parallelogram law, as shown on the right in Figure 3, is used when the vectors to be added have the same origin, in which the geometric image of each of the vectors is positioned at the ends of the vectors, thus forming a parallelogram, the vector sum is given by the diagonal of the parallelogram. In mathematics, adding vectors using the law of the polygon or the law of the parallelogram is the same thing, regardless of the mathematical context that is used, both laws are convenient and represent the same resultant vector.

In Physics and Engineering, on the other hand, the use of the laws is considered depending on the context, that is, depending on what these vectors are representing, it makes more sense to use one law than the other. For example, if the context involves displacement, it makes more sense to use the polygon law, whereas if the context involves the sum of forces, it makes more sense to use the parallelogram law. There is no scientific definition regarding the use of these laws that supports this idea: it is an "artificial" construct used by scholars that has been used for generations.

However, for the procedures of both rules to be carried out through processes of understanding, it is necessary that equipollence and equivalence relations are internalized by the students, since the geometric image we referred to earlier in the parallelogram rule considers the equivalence relation of the vector that is being displaced to form the parallelogram. This happens in several situations of operations with vectors when the vector is presented in its geometric form, or when the geometric representation of the vector is necessary for the operation to be developed correctly.

As already discussed in Roncaglio, Battisti and Nehring (2021, p. 286, our translation):

> Unlike the mathematical context, in Physics and consequently in Engineering there are three types of vectors that are necessary to be defined due to the

> Physical specification and the context in which the vector is being mobilized, that is, they are structuring for understandings in specific contexts of Physics, they are the fixed vector, the sliding vector and the free vector.

It is the relationships established with the concepts that are part of the Vector conceptual system that enable the mobilization of this concept in different situations and in other areas of knowledge. Mathematics has a language that allows the articulation of several registers of semiotic representation and Physics uses this language. For example, the vector language to represent phenomena such as force, displacement, velocity, and acceleration, quantities that, in order to be defined, need a module, a sense, and a direction.

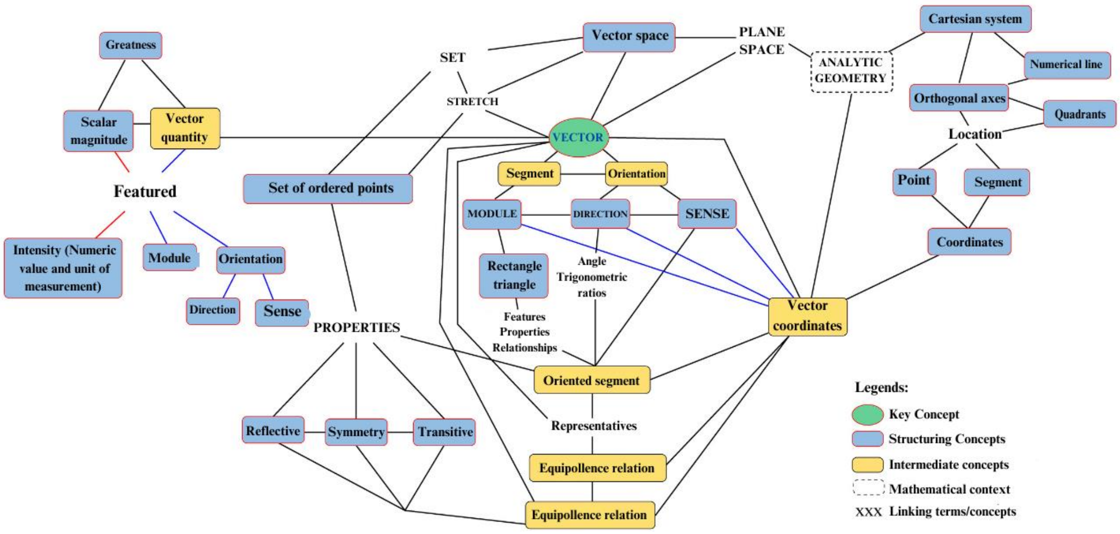
Vector is a mathematical concept used as a language by Physics and Engineering to describe phenomena that, in order to be defined, require a sense, a direction and a module (intensity), usually used to describe a displacement or situations involving vectorial quantities, as is the case of the vectorial quantity force. A vector quantity is defined by its sense, module and direction, elements that make up the vector, which need to be meant in the teaching and learning process and that mark the scalar quantity delimiter and the need for a vector quantity.

> The language of vectors lends the concept of force its structure so that it can be defined as a physical concept. The arrow placed above the letter indicates that it is a quantity with direction and meaning. The vector language has its own grammar, syntax and spelling, which are the axioms, theorems, lemmas, rules of application, etc. A concept such as Force, when identified with a vector quantity, is subject to all its language rules. It becomes difficult to express it in any other way, for example, through ordinary written language. Textbooks reflect the difficulties of this endeavor, being a kind of dictionary between the common language and the language of vectors.
> Mathematics is one language among many others at our disposal to structure our thinking. It has proven, over the centuries, its exceptional ability to support our thinking about the world (PIETROCOLA, 2002, p. 105, our translation).

Thus, we highlight the importance of mathematics as an area of knowledge responsible for providing intellectual tools that allow the description and analysis of problem situations in various areas, especially in the area of engineering, our focus of study. And in this context, the concept vector as an essential concept for the professional education of the engineer. In this sense, considering mathematics as a science that has a language that allows the subject to structure thought in order to appropriate the knowledge built by man, the teaching and learning process must provide the development of this ability.

Considering the process of teaching and learning the mathematical concept Vector, it is understood that this process should articulate the relationships that involve this concept, such as the elements of vector formation, modulus, direction and sense. To signify these elements of formation, as well as line segment, equipollence relation, among others that relate to vector, through the attribution of meanings, enable students to appropriate a network of generalization involving various mathematical concepts. This relationship can be observed in the concept map developed by Battisti and Nehring (2020), presented below.

Figure 4 – Concept map of the vector concept



Source: Battisti and Nehring (2020, p. 133)

As the map shows, there are numerous relationships that can be performed considering the concept Vector, the Vector's formation elements have specificities and are also inserted in a network of generalizations.

> It is the generality relations that will enable the scholar a conceptual elaboration, as well as the evolution in the levels of this conceptualization. Appropriate the meaning of scientific concepts through processes of analysis (abstraction) and synthesis (generalization) implies that the student places the concept vector in a given network of meanings, and also considers that each concept has specific structures, with specific thoughts and reasoning (BATTISTI; NEHRING, 2020, p. 132-133, our translation).

Davis and Hersh (1985, p. 173) contribute to the discussion by pointing out that: "A mathematical object considered in isolation has no meaning. Its meaning comes from a structure, and it represents its role within a structure". Thus, we highlight the importance of considering these relationships in the process of teaching and learning Vector, an aspect that holds true for the process of teaching and learning any mathematical concept.

Final remarks

This study presents a theoretical contribution in relation to the concept of Force, from the definitions presented in chapters of books indicated in the basic bibliography of subjects of Physics I and General Mechanics I: the understanding of the concept Force as a vectorial magnitude, considering the field and conceptual relations that constitute it. From this, we conclude that: the discipline General Mechanics I is the first specific discipline of the Engineering Courses that mobilizes the Vector and the Force. This subject requires students to have clear basic concepts, such as Vector and Force, that is, it is a subject that discusses situations in the Engineer's professional field by means of situations involving the design of beams, loads, and the analysis of structures, among other situations. In this subject the students need to mobilize theoretical knowledge from Mathematics, regarding the application of the concept Vector learned in Analytic Geometry and Vectors, and from Physics, regarding the application of the concept Force studied in Physics I.

However, for students to grasp the basic concepts of mechanics, it is necessary that they mobilize the physical concept of force, a central concept in the subject General Mechanics I. The subject Physics I is responsible for introducing the concept of Force, defining and characterizing it as a necessary and important concept applied in several specific disciplines of Engineering Courses. As we saw in the definitions presented in relation to the concept of force in chapters of the books of the basic bibliography of Physics I, they present the concept as a vectorial quantity, because it needs a module, a direction and a sense to be defined. The concept Force as a vector quantity requires the use of a vector for its representation, be it algebraic and/or geometric.

The Vector concept is introduced in the courses in the first semesters, usually in the first or second semester as a basic subject, it is a subject that works with one of the structuring concepts of the course and serves as a basis for other specific subjects. The concept Vector arises to represent magnitudes that cannot be defined only by a numerical value, but that depend on other characteristics, such as modulus, direction and sense. In Engineering courses there are many magnitudes that need these characteristics to be represented, and the magnitude force is one of them, one of the main ones.

Vector is the concept that allows working with this and several other vector magnitudes, mobilized in several areas. However, for this concept to be effectively mobilized, there are some relations that need to be considered, especially when they involve operations with these magnitudes, relations such as equipollence, equivalence, and other constituent concepts of the vector, as we can see in Figure 3. It is a mathematical concept that belongs to a network of concepts, which are related to each other, providing support and structure to the vector: it is from the mathematical properties that all relations involving this concept are explained.

The appropriation of the concepts Vector and Force constitute cognitive tools necessary for the development of competencies and skills related to the engineer's field of action. They are concepts that, like several others, are strongly articulated. Mechanics requires the concept of force to explain and represent situations, problems and solutions: force is a vectorial quantity and as such requires the concept vector to be represented and the operations with vectors to explore situations, solve problems and find solutions.

In this sense, from the analyses performed, we can say that the approach given to the concept Force is strongly related to a vector quantity and to intuitive notions of some kind of muscular force, such as pushing or pulling. And, in this context, the concept Vector is approached to represent geometrically and/or algebraically, situations that involve force. However, it should be noted that in the mathematical context Vector is understood as a free vector, since it can be represented anywhere in the plane or in space by means of its representatives. In Physics and Engineering the vector can be understood as a sliding vector, when it involves forces that are applied along a support line, or as a fixed vector when the context involves the application of a point force.

In the course of the analyses performed, it is possible to perceive the idea of conceptual relations that involve the concept discussed here, the concept of force. Considering the theory that underlies this study, Cultural Historical Theory, which argues that a concept is not formed alone, but in relationships with other concepts, it became evident during this writing, from the concept maps presented during the analysis, the conceptual relationships attributed to the concept of force.

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