INITIAL AND CONTINUING TEACHER EDUCATION: THE DEVELOPMENT OF PROFESSIONAL LEARNING TASKS FOR THE STUDY OF THE CONCEPT OF FUNCTION

FORMAÇÃO INICIAL E CONTINUADA: A ELABORAÇÃO DE TAREFAS DE APRENDIZAGEM PROFISSIONAL PARA O ESTUDO DO CONCEITO DE FUNÇÃO

FORMACIÓN INICIAL Y CONTINUA: LA ELABORACIÓN DE TAREAS DE APRENDIZAJE PROFESIONAL PARA EL ESTUDIO DEL CONCEPTO DE FUNCIÓN

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ABSTRACT: This article aims to present the Professional Learning Tasks elaborated on the concept of function as a resource to enable professional learning opportunities linked to mathematical and didactic knowledge for teachers and future teachers of mathematics. For the elaboration of the tasks was chosen the model Professional Learning Opportunities for Teachers, that uses the components of the conceptual dimension established based on the fundamental concepts and learning aspects of the content of functions, defined by the material to support the work of teachers. In this regard, the training instrument developed indicates elements of structures and possibilities for the initial and continuous development of teacher learning grounded in practice, whereas, for this article does not present empirical results, because there was no data collection, only the preparation of tasks.

KEYWORDS: Concept of function. Professional learning tasks. Mathematical tasks. PLOT Model. Teacher's knowledge.

RESUMO: *O presente artigo visa apresentar as Tarefas de Aprendizagem Profissional elaboradas sobre o conceito de função como um recurso para possibilitar oportunidades de aprendizagem profissional vinculadas aos conhecimentos matemáticos e didáticos dos professores e futuros docentes de matemática. Para a elaboração das tarefas foi escolhido o modelo Professional Learning Opportunities for Teachers, utilizando as componentes da dimensão conceitual estabelecidas com base nos conceitos fundamentais e nos aspectos de aprendizagem do conteúdo de funções, definidos pelo material de apoio ao trabalho dos professores. Nesse aspecto, o instrumento formativo elaborado indica elementos de estruturas e possibilidades para o desenvolvimento inicial e contínuo da aprendizagem do professor fundamentada na prática, sendo que, para este artigo, não se apresenta resultados empíricos, visto que não houve coleta de dados, apenas a elaboração das tarefas.*

PALAVRAS-CHAVE: *Conceito de função. Tarefas de aprendizagem profissional. Tarefas matemáticas. Modelo PLOT. Conhecimento do professor.*

RESUMEN: *Este artículo tiene como objetivo presentar las Tareas de Aprendizaje Profesional elaboradas sobre el concepto de función como recurso para posibilitar oportunidades de aprendizaje profesional vinculadas al conocimiento matemático y didáctico de los docentes que imparten clases de matemática y futuros docentes de esta disciplina. Para la elaboración de las tareas fue elegido el modelo Professional Learning Opportunities for Teachers, que utiliza los componentes de la dimensión conceptual establecidos sobre la base de los conceptos fundamentales y los aspectos de aprendizaje del contenido de funciones, definidos por el material de apoyo al trabajo de los profesores. En este aspecto, el instrumento formativo elaborado indica elementos de estructuras y posibilidades para el desarrollo inicial y continuo del aprendizaje del profesor fundamentada en la práctica, este artículo no presenta resultados empíricos, pues no hubo recolección de datos, únicamente la elaboración de las tareas.*

PALABRAS CLAVE: *Concepto de función. Tareas de aprendizaje profesional. Tareas matemáticas. Modelo PLOT. Conocimiento del profesor.*

Introduction

Initial teacher education takes place in undergraduate Teaching and Pedagogy courses and through other extracurricular activities that make up moments of teaching learning. Such learning cannot be dissociated from initial and continued education, since pedagogical practice represents the element that interconnects teacher education processes through theoretical discussions and practical exercises – which expresses the inseparability of the different periods of teacher education (PIMENTA, 2012; TARDIF, 2014).

In this aspect, studies focusing on the connection between initial and continuing teacher education have been little carried out (ROLDÃO, 2007), so they do not articulate both fields of investigation, which diverges from the conception of professional development as a *continuum*[3] (NÓVOA, 1991). The initial education of future mathematics teachers involves developing knowledge and conceptions regarding mathematical concepts and pedagogical knowledge for teaching. Thus, given that teaching professional development is based on continuous education, continuing education involves teaching experience – so that teachers who teach mathematics can improve their repertoire of content and student knowledge – and helps in overcoming evident obstacles in the classroom environment (ZUFFI; PACCA, 2002).

Research by Gorzoni and Davis (2017) and Rangel, Giraldo and Maculan Filho (2015) highlights the encouragement and proposal of collaborative environments in which teachers can reflect on their practice in light of a theoretical framework, so that the education process is not isolated. It is a way to establish collaboration throughout teacher education and the use of Professional Learning Tasks (TAP) (BALL; COHEN, 1999; SMITH, 2001; SWAN, 2007), which explain artifacts of practice, such as curricular materials, videos, class episodes and student work.

TAP, according to the conceptions of professional learning by Ball and Cohen (1999), indicates that collective discussions are the basis for professional learning, because, through dialogue, teachers can expand their own opportunities to learn by understanding, comparing and (re)formulate their own uncertainties. The use of TAP as a pedagogical didactic resource for the initial and continuing education of teachers focuses on three pillars: the artifacts of practice, the developments based on said TAP and the role of the teacher trainer — mediator of professional learning opportunities and new teaching practices throughout the education process.

[3] According to Nóvoa (1991), *continuous* development implies the ongoing training of the teacher, so that a continuous process of professional growth occurs, which involves initial and continued training.

With the exploration of specific mathematical content to be implemented in TAP, this article focuses on the presentation and discussion of TAP designed to study the concept of function in the initial and continuing education of mathematics teachers . From this perspective, the link between the student and knowledge is established through the teacher [4], thus, in addition to helping to understand the knowledge that is present in the students' thoughts, anchored in learning in practice and with practice, the use of TAP can enable the construction of knowledge in an active way, in order to establish the most diverse relationships (imbalances, new hypotheses and interpretations) that involve the concept of function.

Theoretical foundation

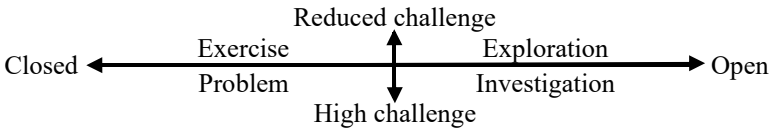
Mathematical Tasks (TM) and Professional Learning Tasks (TAP)

The development of the TAP will be supported by research by Ponte (2014), Ball and Cohen (1999) and Silver *et al*. (2007) with regard to its structuring. In the classroom context, the word "task" can be understood as a product usually prepared by the teacher, but not necessarily. Ponte (2014) with the purpose of mobilizing knowledge and focusing students' attention on a mathematical idea (STEIN *et al*., 2009), which in turn refers to a mathematical activity to solve (CUNHA, 2000; PONTE, 2014). Therefore, the task is an action external to the student and the activity is an action carried out by him.

According to Boavida *et al*. (2008), the teacher can use different types of tasks when considering the learning context, whether those that lead to memorization and procedural practice or those oriented towards more elaborate thoughts. In this regard, Ponte (2005) points out four different types of tasks: the exercise, the problem, the exploration and the investigation, as suggested in Figure 1. The horizontal structure refers to the degree of clarity of the content, that is, the types of reasoning required divided between open and closed, while the vertical structure is related to the perception of the difficulty of the task, and the levels of cognitive demand vary between low and high.

[4] The aforementioned relationship between teacher - knowledge - student refers to D'Amore's triangle of didactics (2007).

Figure 1 – Relationship between types of tasks, in terms of their degree of challenge and openness



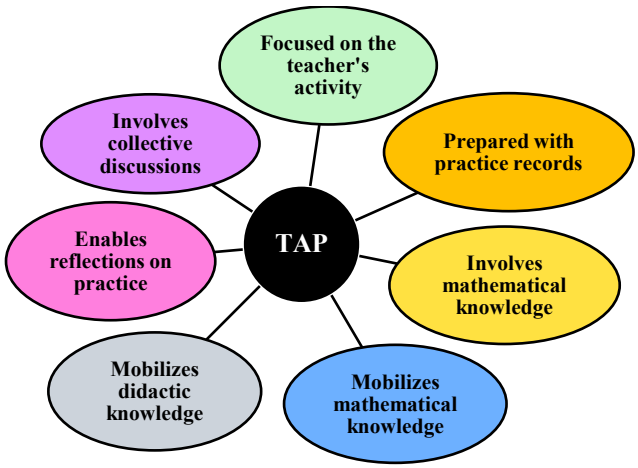
Source: adapted from Ponte (2005)

This relationship between different types of tasks helps to classify Mathematical Tasks (TM), understood as

> [...] tasks that ask students to perform a memorized procedure, in a routine way, represent a certain type of opportunity for students to think; Tasks that require students to think conceptually and encourage them to make connections represent a different type of opportunity for students to think (STEIN; SMITH, 2009, p. 22, our translation).

On the other hand, with the aim of expanding the teacher's domains of knowledge (BALL; THAMES; PHELPS, 2008), TAP represent an opportunity for the teacher's professional learning (BALL; COHEN, 1999; RIBEIRO; PONTE, 2020; SILVER *et al* ., 2007), as they add elements that provide improvement: (Figure 2).

Figure 2 – Structure and possibilities of TAP for teacher education



Source: adapted from Barboza (2019)

Regarding the elements listed by Barboza (2019), it is highlighted that these tasks are focused on the teacher's activity; involve mathematical concepts; mobilize mathematical and didactic knowledge; they cover discussions that can enable reflections on teaching practice and, finally, are prepared with records of practice[5]. Thus, the TAP in this article are composed of

[5]They involve artifacts of practice, such as curricular materials, videos, class episodes and student work (BALL; COHEN, 1999).

TM, intended for Basic Education students, so that they represent records of the practice to be carried out by the students.

Other components that can provide opportunities for professional learning for teachers refer to the role and actions of the trainer (RIBEIRO; PONTE, 2020), so that the "interaction between teachers and the trainer, in the use of TAP, can be an important factor promoting learning opportunities, based on practice" (BARBOZA; PAZUCH; RIBEIRO, 2021, p. 7, our translation). TAP also allows you to visualize the effects of the trainer's planning on the construction of the teacher's mathematical and didactic knowledge.

The study of functions in teacher education

Current and more complex mathematical concepts are elaborated through continuous evolution, a composition generated in different historical periods by different human minds (ZUFFI; PACCA, 2002). The concept of function, for example, had its first formal elaboration from the studies of movements and rates of change of continuously varying quantities, developed by Newton (1642-1727) and Leibniz (1646-1716) (SOUSA; MOURA, 2019).

However, the evolution of the concept of function did not stagnate in the works of Newton and Leibniz, on the contrary, it expresses meanings beyond the understanding of natural phenomena, since, in Mathematics, this concept applies to generalizations, to problem solving and the formalization of other more abstract mathematical concepts (SOUSA; MOURA, 2019). Regarding the teaching of Functions in Brazilian territory, the curricular guidelines on what "should" be taught by mathematics teachers to their students in Basic Education are organized and structured through the National Common Curricular Base (BNCC), already what

> [...] curricula have complementary roles to ensure the essential learning defined for each stage of basic education, since such learning only materializes through the set of decisions that characterize the curriculum in action (BRASIL, 2017, p. 12, our translation).

In relation to the Mathematics curriculum, at BNCC the Functions content is presented in the field of Algebra. This document defines the set of essential knowledge and skills that students are expected to develop in the learning process.

Understanding mathematical concepts linked to the field of Algebra requires the development of algebraic thinking, which allows "dealing with other mathematical relationships and structures and using them in the interpretation and resolution of mathematical

problems or problems in other domains" (PONTE; BRANCO; MATOS, 2009, p. 10, our translation). For Ribeiro and Cury (2015), algebraic thinking represents a process of generalization carried out by students, whose arguments presented are mathematical ideas arising from observations of a set of particular data.

Thus, Ribeiro and Cury (2015) revisit the concepts of functional thinking and refer to it through the act of generalizing the variation of two related quantities and exploring the fundamental idea of the mathematical concept of function. Therefore, the configuration of Function teaching as a dimension of Algebra reinforces the articulation between the fields of knowledge, determined by equivalence, order, proportionality, interdependence, representation, variation and approximation (BRASIL, 2017).

The construction of the concept of function and the respective knowledge linked to it begins in Basic Education, and is expanded and formalized in initial teacher education. However, it is necessary to continue the construction of the concept through continuing education processes, given that much of the professional knowledge emerges in practice (LAMPERT, 2010). The teacher plays a significant role in involving mathematical concepts, curricular, pedagogical and didactic aspects in his practice, since "professional knowledge is built in action and interaction and is only really useful if it can be mobilized in action" (PONTE, 1999, p. 16, our translation).

With this, it is understood that the teacher's learning continues and is based on the practice of teachers who are already in the classroom work environment and future teachers, who (re)signify their knowledge when coming into contact with practical experiences professionals. The working proposal of the concept of function with the use of TAP reinforces the perspective of learning in practice and with practice, of investigating teaching in teaching itself, in order to provide opportunities for professional learning, whether in initial or continued education.

Methodological procedures

Preparation of a reference framework

To develop a reference framework, we chose to use the *Professional Learning Opportunities for Teachers* (PLOT) model[6], developed by Ribeiro and Ponte (2020) for planning and implementing teacher education programs. This model was established by researchers to unify three domains present in teacher education research in the field of Mathematics Education that involve the Role and Actions of the Trainer (PAF), TAP and Discursive Interactions between Participants (IDP).

The PLOT model, by associating teacher learning opportunities in its structure, aims to conceive an interactive and interconnected perspective to teacher education processes (RIBEIRO; PONTE, 2020). The structure of each domain of the PLOT model is composed of four components divided between the conceptual dimension, characterized by the structure and theoretical bases, and the operational dimension, which expresses the guidelines for how to apply the model, as can be seen in Table 1 The joint action of both dimensions is aimed at organizing a teacher education process, identifying and understanding how professional learning opportunities arise for teachers and whether they appear (RIBEIRO; PONTE, 2020) .

Table 1 – Dimensions, components and characteristics of the PLOT model in three domains

	Conceptual dimension		Operational dimension	
	Component	**Feature**	**Component**	**Feature**
Role and Actions of the Trainer (PAF)	***Approximation***	Favor the approximation of Academic Mathematics (MA) to School Mathematics (ME) and vice versa.	***Management***	Promote the management of an exploratory teaching-learning environment, with the different phases of this approach.
	Articulation	Stimulate the articulation between the mathematical and didactic dimensions of professional knowledge for teaching.	***Orchestration***	Prepare and develop the orchestration of mathematical and didactic discussions among all participants.
Professional Learning Assignments (TAP)	***Professional knowledge***	Explore teachers' mathematical and didactic knowledge related to TME/s.	***Math Task***	Contemplate students' mathematical task/s (TME), with a high cognitive level.
	Exploratory Teaching	Have a structure that provides an exploratory teaching-learning environment.	***Practice Records***	Involve different types of practice records, organized in the form of *Vignettes* .

[6]The PLOT model was designed to meet the demand for the development of shared structures for the study of teacher learning through an interactive and interconnected perspective.

Discursive Interactions between Participants (IDP)	***Discussions Mathematics and Didactics***	Contemplate, in an articulated way, mathematical and didactic discussions related to TME.	***Language mobilized***	Consider the use of appropriate mathematical and didactic language relevant to the teaching level of the TME.
	Argumentation and Justification	Involve valid mathematical and didactic argumentation and justification.	***Communication dialogical***	Promote dialogical and integrative communication between all participants.

Source: Ribeiro e Ponte (2020)

In this article, the TAP domain with its respective components was exclusively considered, approaching the conceptual dimension through professional knowledge and exploratory teaching and the operational dimension through mathematical tasks and practice records.

Table 2 – Dimensions, components and characteristics in the TAP domain with a focus on teaching the concept of function for different teaching levels in Basic Education

	Conceptual Dimension		**Operational dimension**	
	Component	**Feature**	**Component**	**Feature**
Early Years of Elementary School	***Professional knowledge***	Work with sequences of natural variable functions.	***Mathematical task***	Identification and tracking of a sequence pattern.
	Exploratory teaching	Each number (order) corresponds to a given term, which can be a number, a geometric object or any other object.	***Practice record***	Mathematical task to be provided for Early Years students.
Final Years of Elementary School	***Professional knowledge***	Correspondences between two variables, represented in tables, graphs and diagrams. The formal definition of the concept of function is not addressed .	***Mathematical task***	Solving a problem focusing on the variation in the area of a polygon as a function of a given distance.
	Exploratory teaching	Emphasis on solving problems related to situations of direct proportionality, which involve functional relationships.	***Practice record***	Mathematical task to be provided to Final Year students, with a script, for constructing the polygon of the problem in the Geogebra software .
High school	***Professional knowledge***	Addressing the concept of function explicitly, in addition to the transition between different representations of a function.	***Mathematical task***	Domain and image analysis of a function from its graph and algebraic expression.
	Exploratory teaching	Students can: - Define what a function is	***Practice record***	Mathematical task to be provided to high school

	(concept and notion [7]); - Identify relationships defined as functions and non-functions; - Indicate domain, image and range; , pattern generalization, graphic and formal representations; - Use the concept in problem solving and modeling real situations.	students, with two incorrect solutions most common among students.

Source: Prepared by the authors

Table 2 presents the structuring of TAP with a focus on teaching the concept of function for each level of teaching in Basic Education, summarizing curricular and didactic directions for teaching. The characterization of each component was established through the analysis of the chapter "Functions", from the work *Álgebra no Ensino Básico* [8], by Ponte, Branco and Matos (2009). According to the conceptual foundations and learning aspects, presented in Ponte, Branco and Matos (2009), the professional knowledge to be mobilized in action and the directions of exploratory teaching were determined, thus, the characteristics of the operational dimension components were stipulated according to the components of the conceptual dimension.

Preparation of a reference framework

Totaling three TAP, they are identified as: TAP – EFI, which is made up of two TM (TM1 and TM2) for the Initial Years of Elementary School; TAP – EFF, which has a TM (TM3) for the Final Years of Elementary School; and TAP – EM which presents TM4, whose mathematical knowledge is linked to the High School curriculum.

Following the guidelines in Table 2, TMs were selected and adapted whose objectives are linked to the characteristics of the components of the conceptual dimension and, based on the use of the colors presented in Figure 2, Table 3 represents a legend for the TAP elements present (directly or indirectly) in its structure.

- Objective(s) TM1: Identify and develop the color sequence pattern of a centipede;
- Objective(s) TM2: Identify and analyze the pedestrian sequence pattern through an image;

[7] Relationship between variables and unique correspondence between two sets.
[8] Material to support teachers' work within the scope of the Basic Education Mathematics Program.

- Objective(s) TM3: Analyze the variation in the area of the BCDP polygon, as a function of the distance from the segment AP = x, in order to determine the graph that describes this behavior;
- Objective(s) TM4: Analyze the domain and image of the function f(x) = ax+3 based on its algebraic expression and its graph.

Table 3 – Color legend of TAP structure elements and possibilities

Element color				
TAP structure and elements	Teacher activity	Involvement of mathematical concepts	Mobilization of didactic knowledge	
Element color				
TAP structure and elements	Practice Records	Mobilization of mathematical knowledge	Possibility of reflection	Collective discussion

Source: Prepared by the authors

Regarding its structure, the TAP were divided into two parts: Part 1 composed of a practice record (TM and/or student record), and in this it is up to the trainer to provide guidance regarding the mathematical and didactic knowledge that can be mobilized in this section; Part 2 refers to the teachers' practice, and the analysis in Part 1 is carried out in parallel to the discussions and reflections mediated by the teacher trainer. Discussions are essential for changes to occur in mathematical communication with regard to the study of a concept through its definition (TABACH; NACHLIELI, 2015), and it is important to emphasize that not all discussions present the potential for a relative change in practice (BARBOZA; PAZUCH; RIBEIRO, 2021; TABACH, NACHLIELI, 2015).

The discussion develops from the presentation of the prepared TAP, which are considered materials for data collection. This article will not describe the process of implementing a TAP in the classroom. Therefore, there will be no inclusion of empirical elements, despite representing a potential resource for developing collaborative education moments.

Discussion and presentation of TAP

The TAP are presented in two figures, Part 1 (Figure 3) referring to TM and Part 2 (Figure 4) with questions for analysis and reflection. It is noteworthy that they can approach the concept of function in different ways, depending on the discussions and reflections carried out by teachers and future teachers and the action of the teacher trainer (BARBOZA; PAZUCH; RIBEIRO, 2021; RIBEIRO; PONTE, 2020).

Figure 3 – Part 1: TAP – EFI / TAP – EFF / TAP – EM

TM1: Centipede Colors

Required material: Modeling clay, piece of string (or similar) and beads or beads of different colors.
Development:

Step 1: Create a centipede with modeling clay so that students, in a circle, identify how the colors are repeated on the centipede's body, that is, the pattern of the sequence. Then, ask students to continue developing the pattern, enlarging the centipede's body.
Step 2: In pairs, students create centipedes with a repeating motif for their partner to discover and continue.
Step 3: Children will be able to record one of the sequences they created.
Source: ADAPTED from http://www.educacao.pe.gov.br/portal/

TM2: Pedrinho's antics

Necessary material: You can provide objects to students so that they represent spinning tops.
Development:
Pedrinho is a very smart and playful boy. He also likes to invent mysteries for his friends to discover. Look at the idea he had:

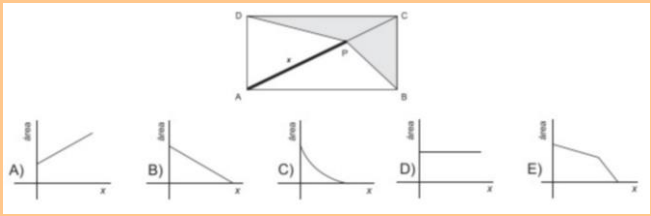
Present the questions below to students so that they can comment and record conclusions regarding Pedrinho's idea:
a) In these images, what do you observe?
b) Are all spinning tops the same?
c) Are the tops repeated in any order? What did you discover?
d) What was this secret that Pedrinho used?
e) Using the secret you discovered, what would be the next figures?

Source: ADAPTED from the SBEM Collection (Volume 12, 2018)

TM3: OBMEP 2007 QUESTION:
Which of the graphs below describes the variation in the area of the BCDP polygon as a function of the distance x=AP?

To carry out this mathematical task, the dynamic software Geogebra will be used . Below is the script for the construction of the BCDP polygon, prepared by a professor:

Source: http://www.obmep.org.br/provas.htm



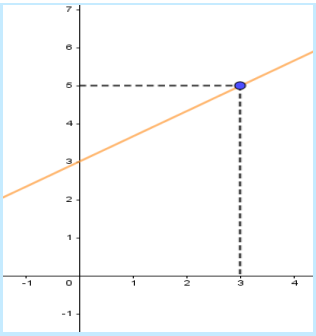
ROAD MAP:

Students must work in pairs or trios to complete the construction and then explore the construction and record their conclusions to find the answer.

- Create points A and B, on the same horizontal, and then determine the segment AB.
- Draw two perpendiculars to the segment AB, through points A and B.
- Mark a point C on the perpendicular through B, using the "Point on object" tool.
- Through C, draw a perpendicular to the line BC and create point D, with the "Intersection of two objects" tool, at the intersection with the other perpendicular.
- Hide the straight lines and construct the segments BC, CD and DA.
- Build the diagonal AC and insert a point P on it.
- Using the "Polygon" tool, build BCDP: click on B, C, D, P and again on B to close the cycle.
- When right-clicking on the polygon, in "Properties", change to "Display Label – Value", to display the numerical value of the polygon area on the screen.
- Build the AP segment and name the legend "x" and display it (through the "Properties" menu).
- Click on Viewing Window 2 (so that the point Q, to be constructed, appears in this new window) and enter the following command in the input bar: Q = (n, in1), where n is the name of the AP segment and pol1 is the identification of the BCDP polygon area.
Note : check the Algebra Window to make sure the name is correct.
- Move point P in Window 1 and check point Q moves along a trajectory in Window 2.
- With the right button on point Q, click on "Enable Trace" and with the right button on point P, click on "Animate".

TM4: Multiple Choice Test

Let be a function with real domain and range given by f(x) = ax+3. What is the value of f(9)?
a) 6
b) 9
c) 10
d) 15
e) 18



Below are the two most frequent types of incorrect student resolutions:

A1:	A2:
f(x) = ax + 3	f(x) = ax + 3
f(3) = 3a + 3	f(9) = a. -3 + 3
f(3) = 3.5 + 3	f(9) = a
f3 = 15 + 3 = 18	f = 9/a
f = 18/3	f = 9
f = 6	

Source: Adapted from Bortoli (2011, p. 45-46)

Figure 4 – Part 2: TAP – EFI / TAP – EFF / TAP – EM

Part 2: Analysis of Mathematical Tasks (TM1, TM2, TM3 and TM4)

1) Solve the proposed tasks and record all the procedures used.

2) For which year would you indicate the application of TM1, TM2, TM3 and TM4? What is being studied (content)?

3) What difficulty (s) might students present when carrying out this task?

4) What mathematical objective(s) can be considered for the use of TM1 and TM2?

5) Considering what you answered in the previous items, would you use this task in a mathematics class? How can they develop it? Would you make changes/adaptations?

Exclusive analysis for TM3)

6) Considering the items below:
- Present correspondences between two variables that can be represented in tables, graphs and diagrams.
- Provide problem resolution.

Taking the items as teaching objectives, how could such objectives be worked on in the classroom?

7) In another perspective of approaching the problem, the teacher has the file Obmep_2007.ggb, represented in the Figure below.

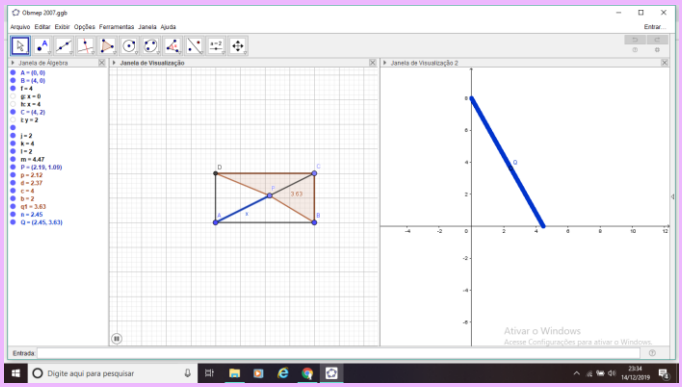


Figure - Obmep_2007.ggb. / Source: the author.

7a) How do you evaluate the script prepared by the teacher in terms of students' understanding of the step-by-step instructions presented?

7b) How would you use the Obmep_2007.ggb file in your classes?

7c) How can they approach and discuss solving the problem if the script was not provided to the students and they had to use Geogebra to find the solution?

Exclusive analysis for TM4)

6) Explain what could have led students to make such mistakes in their answers? Comment and give feedback on the wrong answers, pointing out the similarities and differences between the resolutions.

	Explanation of incorrect answers	**Similarities**	**Differences**	**Return**
R1				
R2				

7) How can the use of technological resources, more specifically Geogebra, help in solving the problem and understanding the definition of function?

Source: Prepared by the authors

TAP– EFI allows the use of diverse manipulable materials, such as modeling clay, logic blocks, Cuisenaire bars, plug-in blocks, various collections, among others. The resource of manipulable materials can be used to physically compose sequences. According to Murari (2011), the use of these resources allows for a more dynamic movement when accompanied by discussion between students and teachers, and the visual representation — of sequence patterns, in this case — is the illustration of the resolution of a problem.

TM1 and TM2 in TAP–EFI work with sequences through the arrangement of a centipede's body and the position in which pawns are located. The development of algebraic and functional thinking is favored considering that the activities carried out by students enhance the ability to *recognize patterns in a sequence – through the perception of their regularity, through the continuation of a repetitive sequence, through the correspondence of a sequence of objects to a given position* – and the analysis of their mathematical discourse, which is revealed throughout the teaching and learning process (RIBEIRO; CURY, 2015).

TAP – EFF establishes a *connection between the content of Function in the field of Algebra and that in the field of Geometry*, as *it lists historical aspects* of the emergence of the concept of function (GONÇALVES, 2015; SOUSA; MOURA, 2019), without explicitly addressing its definition (PONTE; BRANCO; MATOS, 2009). The proposal to use the Geogebra tool to solve TM3 also provides the connection between Algebra and Geometry by "relating the information given algebraically with graphical and table representations and presenting mathematical objects in a representation closer to the usual one" (PONTE; BRANCO; MATOS, 2009, p. 16, our translation).

Through the analysis of TM3, proposed by TAP – EFF, teachers can be encouraged to discuss the understanding of the concept of function through different representations, such as the transition between Cartesian graphics, algebraic language and geometric language. By linking the objective of *associate the concept of functions with the study of the properties of different representations*, it is necessary that the concept (of function) and its integrative character (to various representations) are understood by those who will teach it, and developed in different ways within the classroom (GONÇALVES, 2015).

TAP *provide opportunities for reflection on pedagogical problems.* TAP–EM brings together discussions regarding students' incorrect resolutions, with the aim of characterizing the origin of the difficulties in conceptual understanding that led the student to error. To understand TM4, it is essential to understand the definition of the concept of function to solve the problem and perceive mistakes made by the student. Possible solutions to problems in pedagogical

practice can be highlighted with the use of TAP, as they provide a process of reflection, sharing and construction of collective knowledge (SILVER *et al.,* 2007).

When analyzing the three TAPs that were prepared, it is highlighted that in all TAPs there are questions that ask for the resolution of the MTs and their adequacy to a given year of teaching with the purpose of determining the teacher's mathematical knowledge and didactic knowledge, respectively. From the identification by color of the structures and possibilities of TAP, it is highlighted that due to the color purple, all TAP are subject to collective discussion, depending on the way in which the teacher trainer conducts the teacher's professional learning (BALL; COHEN, 1999; SMITH, 2001).

The records of practices (orange color) express an essential part of TAP's objectives, as they concentrate the theme of possible discussions and the consequent reflections, which are the basis for professional learning (BALL; COHEN, 1999; BARBOZA, 2019; SILVER et *al*., 2007). References to the teacher's activity (green color), the mobilizations of mathematical (blue color) and didactic knowledge (gray color) and the possibility of reflection (pink color) are mostly concentrated in the second part of the TAP, which directs to the analysis of practice records (orange color) and the approach to mathematical concepts through TM (yellow color).

Regarding this, Barboza (2019) reveals that the choice of the TAP structure is central in the teacher's learning process, so that he or she experiences the "formation of new knowledge structures, combining several of its spheres and potentially some new knowledge, whether individually or collectively" (p. 23, our translation). On the other hand, the author highlights that TAP, intrinsically, do not have the capacity to (re)signify and mobilize mathematical and didactic knowledge, and it is essential to recognize the reflections of the teacher who works with them, through questions from the trainer and the discussions triggered.

Considering that the difficulty in teaching and learning functions by students and teachers is verified in the formalization of the concept of function and in the relationships with representations, the elaboration of tasks can represent a challenge (GONÇALVES, 2015). Therefore, listing opportunities for teachers to highlight mathematical objectives and teaching strategies for the application of MT in TAP can expand the possibilities for initial and continuous development of teacher learning based on practice.

Final remarks

This article aimed to *present and discuss the TAP designed to study the concept of function*. In it, the elements of TAP were identified and it was evident that discussions and reflections on the practice occur through the analysis of the record of this practice (BARBOZA, 2019) and through questions whose developments are guided by the trainer (BARBOZA; PAZUCH; RIBEIRO, 2021). The didactic approach to the concept of function is then adapted for each level of education, in search of the gradual development of functional thinking, familiarity with the different representations and the formal presentation of the definition (PONTE; BRANCO; MATOS, 2009).

The attempt to establish a rapprochement between the components of the conceptual dimension and the operational dimension, present in Table 1, through TAP, aims to resolve the demand for shared structures for developing the study of teacher learning. In this aspect, TAPs establish opportunities for the initial and continuing education of teachers by (re)signifying mathematical and didactic knowledge through an interactive and interconnected perspective of practice.

The proposal to develop professional learning in an environment with divergent profiles aims to promote the education of mathematics teachers through task planning, teaching through practice and the production of knowledge (GUMIERO; PAZUCH, 2019). For a greater understanding of professional learning and the effectiveness of using TAP, it is necessary to use them as instruments for collecting empirical data with a heterogeneous group. For this reason, it is noteworthy that establishing strategies for the professional learning of mathematics teachers requires renewals and new alternatives to the teacher's practice, and TAP is a means of jointly reflecting on the choice of resources associated with the objectives and mathematical ideas you want to work with.

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