PROCESS OF APPROPRIATION OF TRIGONOMETRIC CONCEPTS MEDIATED BY THE TEACHING GUIDING ACTIVITY (AOE)

PROCESSO DE APROPRIAÇÃO DE CONCEITOS TRIGONOMÉTRICOS MEDIADO PELA ATIVIDADE ORIENTADORA DE ENSINO (AOE)

PROCESO DE APROPIACIÓN DE CONCEPTOS TRIGONOMÉTRICOS MEDIADO POR LA ACTIVIDAD ORIENTADORA DE ENSEÑANZA (AOE)

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ABSTRACT: Mathematics has historically been perceived as difficult to understand. In Geometry, it has been based on memorizing definitions and formulas, which denies students the possibility of learning the historical logic of the concepts. The aim of this article is to analyze the Teaching Orientation Activity (AOE) as a methodological principle in the process of appropriating trigonometric concepts. It is a cross-section of data from a master's research in Mathematics (BEZERRA, 2019), anchored in the qualitative approach that is inserted in a context of development of AOE with high school students form the state public network of Maranhão, Brazil. It was supported by Activity Theory and the concept of AOE itself. The results showed that AOE is a possibility for organizing the teaching of trigonometric concepts. Under these conditions, it is based on the historical logical movement of the concept, as it enables the student to become aware of the need to find a solution to the triggering problem, placing them in a learning activity.

KEYWORDS: Guiding Teaching Activity. Trigonometric Concepts. High School.

RESUMO: *A Matemática, historicamente, vem sendo significada como de difícil compreensão. Delimitando à Geometria, esta tem sido calcada na memorização de definições e fórmulas, o que nega aos alunos a possibilidade de aprendizagem do lógico histórico dos conceitos. Este artigo tem o objetivo de analisar a Atividade Orientadora de Ensino (AOE) enquanto princípio metodológico no processo de apropriação de conceitos trigonométricos. Trata-se de um recorte dos dados de uma pesquisa de mestrado em Matemática (BEZERRA, 2019), ancorada na abordagem qualitativa, que se insere em um contexto de desenvolvimento de AOE com alunos do Ensino Médio da rede pública estadual do Maranhão, Brasil. Foi subsidiada pela Teoria da Atividade e pelo próprio conceito de AOE. Como resultados, constatou-se que a AOE se apresenta como possibilidade de organização do ensino dos conceitos trigonométricos. Nessas condições, parte-se do movimento lógico histórico do conceito por possibilitar ao aluno a tomada de consciência da necessidade de encontrar uma solução ao problema desencadeador, colocando-o em atividade de aprendizagem.*

PALAVRAS-CHAVE: *Atividade Orientadora de Ensino. Conceitos trigonométricos. Ensino Médio.*

RESUMEN: *La Matemática, históricamente, han sido significadas como de difícil comprensión. Delimitando a la Geometría, esta ha sido fundamentada en la memorización de definiciones y fórmulas, negando a los alumnos la posibilidad de aprendizaje del lógico histórico de los conceptos. Este artículo tiene el objetivo de analizar la Actividad Orientadora de Enseñanza (AOE) como principio metodológico en el proceso de apropiación de conceptos trigonométricos. Se trata de un recorte de los datos de una investigación de maestrazgo en Matemática (BEZERRA, 2019), anclado en el abordaje cualitativa que se inserta en un contexto de desarrollo de AOE con alumnos de la Enseñanza Secundaria de la red pública estatal de Maranhão, Brasil. Fue subsidiada por la Teoría de la Actividad y por el propio concepto de AOE. Como resultados, se constató que la AOE se presenta como posibilidad de organización de la enseñanza de los conceptos trigonométricos. En esas condiciones, se parte del movimiento lógico histórico del concepto por posibilitar al alumno la toma de conciencia de la necesidad de encontrar una solución al problema desencadenador, poniéndolo en actividad de aprendizaje.*

PALABRAS CLAVE: *Actividad Orientadora de Enseñanza. Conceptos trigonométricos. Enseñanza Secundaria.*

Introduction

The low performance of internal and/or external assessments covering the mathematics discipline may imply evidence of the non-production of meanings and, consequently, the non-appropriation of its concepts. Delimiting it to Geometry, in the way it has been worked historically, this is seen by students as difficult to understand and appropriate. On the other hand, based on memorizing definitions, formulas and problem solving, it denies students the possibility of learning the historical logic of concepts. As a result, as Bezerra (2019) highlights, there are many questions we often hear, such as: teacher, why and why study this Mathematics? How did this come about?

In view of the above, we situate the object of study developed by Bezerra (2019) – possibilities of appropriating trigonometric concepts – as we understand that it is the teacher's role to create objective and subjective conditions for students to educate their motives in the search for responding to a given learning trigger situation. To achieve this, it is necessary for the teacher to intentionally organize teaching so that conceptual appropriation occurs. We also clarify that this study, anchored in a qualitative and explanatory approach, was inserted in a context of experience and development of AOE with students in the 2nd year of high school at a state public school in Maranhão.

That said, in this article, we seek to answer the problem question: how does AOE, as a methodological principle, enable mediation in the process of appropriating trigonometric concepts? With this, we aim to analyze the possibilities of AOE as a methodological principle in the process of appropriating trigonometric concepts.

Having made the considerations, in this study the Activity Theory (LEONTIEV, 1978, 2010) and, in particular, the AOE (MOURA, 1996, 2016) present themselves as theoretical-methodological bases that support our investigative and analytical process. AOE is a methodological principle that emerges from the assumptions of Activity Theory. In this principle, Bezerra (2019), based on Leontiev (1978, 2010) and Moura (2001), understands that the activity of learning mathematical concepts must start from a need, driven by a reason, unlike lists of repetitive exercises, with proposals for solving problems in a memorized and mechanical way.

To this end, the sections were organized in the following sequence: in the first, we present introductory considerations about the object of the study, which deals with the appropriation of trigonometric concepts in a context of experience and development of AOE. In the second section, we bring reflections on Activity Theory and AOE, aligned with the issue

of appropriating trigonometric concepts. In turn, in the third section, we explain the methodological procedures that guided the development of this study. Continuing, in the fourth section, we reserve the analysis of the data captured and presented in the learning episode that emerged from the movement of applying the AOE. And, finally, the final remarks, in which we present answers to the research problem.

From Activity Theory to Teaching Guiding Activity: contributions to the process of appropriating trigonometric concepts

Activity Theory constitutes an aspect of Historical-Cultural Theory studies, initiated in the 1930s by the Belarusian Lev Vygotski. His research highlights the interaction of man with signs and instruments of communication, experiences, biological aspects, as well as material and social conditions. As Bezerra (2019, p. 42, our translation) states, these are anchored in the "[...] Marxist assumption that by transforming nature, man also transforms, achieving within objective and subjective conditions his emancipation, his development psychic".

From this perspective, the social and historical activity of men is what triggers the development of their psyche, which occurs through the mediation of instruments and signs based on the appropriation of the physical and/or mental operations that are incorporated in them. Therefore, by appropriating material and symbolic culture, a legacy objectively produced and accumulated throughout history, such as mathematical concepts, man develops. In this sense, when delimiting the pedagogical activity, that is, the unity of teaching activity and learning activity, Bezerra (2019) states that this development is the result of the mediations that permeate it.

In this way, the development of specifically human functions, such as reflection, creativity, imagination, voluntary attention and volitional behavior, originates in the process of appropriating concepts, implying the transformation of external activity into internal activity. Thus, it is necessary for psychic functions to be experienced in the collective, since such functions "[...] do not exist in the individual as a potentiality, but are initially experienced in the form of interpsychic activity (between people) before assuming the form of intrapsychic activity (within the person)" (MELLO, 2004, p. 141, our translation).

In this way, it is evident that men, as a result of their needs and motives, idealize and create social objects, that is, "the product of work contains the human activity fixed in it, it is the objectification of work, it is the activity transformed into an object [...]" (SAVIANI; DUARTE, 2012, p. 23, our translation). In the aforementioned sense, when we focus our

attention on the activity of teaching mathematics, the teacher makes use of mediating instruments, creating conditions for students to grasp mathematical concepts. This involves the need to identify, organize, sequence and measure content, as well as the means to validate the purpose of educational work (SAVIANI, 2011).

It follows from this understanding that it is through activity that men become aware of their actions. Like pedagogical activity, in the case of mathematics teachers, as Bezerra (2019) highlights, they produce meaning of concepts based on their practices. Meaning understood here in the light of Activity Theory as being "[...] the way in which a man assimilates generalized and reflected human experience (LEONTIEV, 1978, p. 101, our translation).

Therefore, consistent with the thoughts of Leontiev (1978), we understand that pedagogical activity enables meanings of the psychological reality of the subject who teaches and learns. From this perspective, the development of this activity generates a change in reality mediated by symbolic instruments, implying objective conditions for the organization of teaching and, consequently, the appropriation of theoretical-scientific concepts by students. Here, therefore, there is a negotiation of meanings, which leads us to understand, as defended by the author in question, that "[...] the concept of activity is necessarily linked to the concept of motive" (LEONTIEV, 2021, p. 123, our translation). From this, we infer that each and every activity is guided by a reason, thus there is a need to educate it.

In light of these reflections, we emphasize that Activity Theory contributes to the educational scenario and, specifically, in the case of this study, to the appropriation of mathematical concepts. However, it is appropriate to highlight the role of the motif as a structuring element of the activity, since it drives the actions of students and teachers, making them aware of their commitment to the pedagogical activity and, consequently, to the organization of teaching. In this regard, we affirm that the actions and operations that determine the activity are impelled by its motive. Furthermore, these are directed towards a purpose (LEONTIEV, 1978).

Still in this regard, with regard to mathematics teaching, conceptual appropriation is mediated by pedagogical relationships involving teacher, student and classmates. Emphasizing pedagogical activity, as Vigotski (2007) says, through the conditions created and mediations, is how the Zone of Imminent Development (ZDI) is formed. This constitutes the possibility for students to reach sophisticated levels of learning, that is, from potential development to real development. In other words, like the problem situations proposed by the teacher, in the first case, it would be what the student carries out, however, with the help of the teacher or a

colleague who demonstrates a more elaborate psychic level. In the second case, it is the level of development at which he can carry out the proposed situations autonomously and creatively.

In view of this theoretical discussion, the question arises: considering the ZDI in the context of mathematical conceptual learning, what theoretical-methodological approaches can we propose to overcome the level of potential development? In the search for answers, Moura (1996, 2001), supported by the assumptions of Activity Theory, idealizes and aims at a methodological principle coined by this author of Teaching Guiding Activity (AOE). This "[...] is structured to allow subjects to interact, mediated by content, negotiating meanings, with the aim of collectively solving a problem situation" (MOURA, 2002, p. 155, our translation).

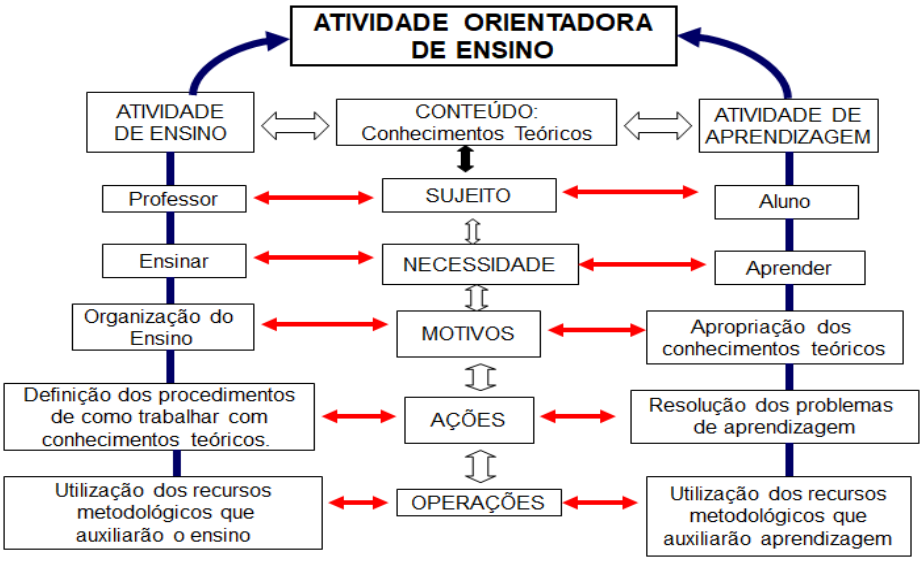
In this direction, based on the logical-historical perspective (KOPNIN, 1978), this principle follows the structure: teacher intentionality; proposal of a situation that triggers learning; moments of interaction between students and the teacher; indication of sources for consultation; possible ways to resolve the triggering problem; and collective discussions that lead to the theoretical synthesis of the resolutions found.

Under these conditions, in the theoretical perspective defended here, we understand that learning only takes shape as an activity when the triad of teacher, student and object of knowledge (in this study, trigonometric concepts) is established through mediation. This understanding presupposes the fundamental role of the teacher in guiding and organizing teaching, with the first action being to create in students the need for the concept. Intrinsic to this action, as previously mentioned, is the need to identify the reasons why students are educated. In line with the above, we emphasize that in the operationalization of actions, the teacher designates methodological principles, such as the AOE, where "[...] teacher and students are active subjects and, in this way, constitute themselves as individuals carrying knowledge, values and affection that will be present in the way they carry out the actions that aim to obtain knowledge of a new quality" (BEZERRA, 2019, p. 36, our translation).

We observe, therefore, that in this proposal for organizing teaching, educational intentions are present, which, certainly, as so well emphasized by Moura (2001, p. 157, our translation), " [...] will provide changes in subjects who acquire new skills, behaviors and values and, in objective reality, as a result of the actions of these subjects".

Having exposed the comments, we summarize the understandings of the AOE by presenting figure 1.

Figure 1 – Structure of the Teaching Guidance Activity



Source: Moura (2016 apud BEZERRA, 2019)

In the description of figure 1, we show that AOE constitutes a mediating axis in pedagogical activity. It contains its characteristic elements: content, subject, need, motives, actions and operations. By way of illustration, we emphasize that in the context of pedagogical activity, the mathematics teacher needs to teach content, such as trigonometric concepts, to students who aim to learn that content. Therefore, this is what justifies the maxim defended by Moura (2001) that AOE constitutes a general way of organizing teaching, whose main content is mathematical theoretical-scientific knowledge and, by extension, the development of theoretical thinking. That said, from the AOE perspective, we have a teaching and learning process that, in short, is constituted by the teacher and student training unit.

Having made these considerations, it is worth highlighting that in the process of elaboration and development of the AOE, three stages are present. In the first, the teacher dedicates himself to studies and planning the historical logic of the central concept (which in the case of this study involves trigonometry). In the second stage, the learning triggering situation (SDA) is created, and the triggering problem must be explained in it. To this end, as a suggestion, the starting point of SDA can be a virtual story (problem situations that involve characters from imagined stories, such as legends and children's stories, as well as stories that contain mathematical concepts), a game or emergent situations of students' daily lives (MOURA, 1996). Finally, in the third stage, the moment of collective synthesis is proposed. Here, under the mediation of the teacher, the negotiation of meanings takes place, seeking to

find mathematically correct answers that coincide with those that humanity, driven by its needs and motives, has historically elaborated and established results as correct (ARAÚJO, 2020).

In view of this theoretical reflection, we highlight in the context of AOE the situation that triggers learning – virtual history –, entitled Verdim and his friends (ROSA *et al.*, 2013), developed by Araújo (2020) when presenting us with a training process with teachers from the Elementary Education, as shown in figure 2.

Figure 2 – Verdim and his Friends

> Once upon a time there was Verdim, an enchanted being who lived in a forest from another world. Verdim had many friends and together they played every day in the forest clearing. Almost everyone lived close to Verdim's house, except three of them: the Giant called Tililim and the two dwarves, Edim and Enim.
>
> One day Verdim invited everyone to play at his house. As Tililim, Edim and Enim lived very far away, Verdim explained how to get to their house. So, leaving the clearing, on the side where the sun sets, they should take fifty steps forward, then thirty steps to the right and another forty steps to the big tree and then they would continue straight ahead and their house would be just ten steps away.
>
> With Verdim's explanation, the three friends wrote down all the instructions so as not to forget anything. The next day, early in the morning, they headed in the right direction. But despite this, they were unable to reach Verdim's house.
>
> *What could have happened? Why didn't they arrive? How can we help Verdim understand what happened and find another way to explain how to get to his house?*

Source: Rosa *et al.* (2013)

The proposal aimed to reach the essence of the concept of measurement, since, for Davydov (1982), this constitutes a general foundation in the development of the concept of number. To this end, initially, meetings were proposed in a dialogical perspective guided by the understanding of the historical logic of measurement.

In the second moment, the teacher presented the virtual story, as shown in figure 2, involving the concept of measurement and its conceptual links, with emphasis on the problem that triggers learning.

Finally, the third moment of the AOE occurred with the negotiation of meanings, so that the answers presented by the teachers participating in the formative study met the perspective of the historical logician of the concept of measurement, coinciding with or, in some way, approaching those meanings established as correct by humanity in the historical-cultural process of development of mathematics.

Therefore, based on the illustration, it is evident that AOE is based on the intentionality of the subjects who, mediated by the negotiation of meanings, enable the objectification and appropriation of theoretical-scientific knowledge and, consequently, the development of theoretical thinking.

Methodological procedures

For the development of this study, in the search to understand the phenomenon investigated, that is, possibilities of appropriating trigonometric concepts, we based ourselves theoretically-methodologically on the Activity Theory and the concept of AOE. To this end, in accordance with Rodrigues and Sforni (2010), we intentionally and collectively created the conditions so that students felt motivated to participate in shared situations. As a result, it was possible for us to follow more closely the verbal manifestations of his thoughts and, by extension, his actions.

Thus, this study is configured as explanatory, given that we resorted to Vygotski's (2007) investigative method, anchored in Historical and Dialectical Materialism. We clarify that the aforementioned method presents three principles: 1) we do not limit ourselves to descriptions of the phenomenon investigated, but rather, we explain this phenomenon, in order to reach its essence; 2) we resort to analyzing the process, the movement of constitution of the phenomenon, that is, its historicity; 3) when faced with the phenomenon being investigated, we must not disregard behaviors that manifest themselves in their crystallized, fossilized appearance, which leads us to the need to return to the genesis of this phenomenon.

In this logic of thought, initially, in order to explain the phenomenon in its essence, we elaborate and present the AOE as a methodological principle. Therefore, we analyzed the historical logical movement of trigonometric concepts and, under the intervention of the researcher, two situations that trigger learning were proposed. Finally, considering the process of appropriating concepts, collectively, we seek to recognize the meanings developed by students.

Given the above, these assumptions signaled elements that allowed us to highlight the movement of the AOE in the objectification and appropriation of trigonometric concepts by high school students. It is worth highlighting that this movement appeared dynamic, historical, constituting an activity, both for students and researchers. Furthermore, we observe that such objectification and appropriation, according to Araújo (2020 apud BEZERRA, 2019, p. 121,

our translation), "[...] is not placed in the social environment, at school or in any other training space, therefore, it is not we just need to describe the process of appropriating mathematical concepts [...]".

The participants were 15 (fifteen) students regularly enrolled in the 2nd year of high school. We clarify that, in order to preserve their identities, it was agreed that we protect their names. Therefore, throughout the text, we adopted code names: Student 1, Student 2, Student 3, Student 4, ...

Specifically, regarding the data production process, in accordance with Oliveira (2016), we understand that the definition of research instruments and techniques must be in line with the problem, the objective of the study and the theoretical-methodological framework, so that they enable the unveiling of the phenomenon investigated. Thus understood, we prioritize participant observation and AOE itself.

Regarding the relevance of participant observation for this study, the choice was made because it is a technique that allows the researcher greater involvement in research actions. As Marconi and Lakatos (2017 apud BEZERRA, 2019, p. 48, our translation) point out, the researcher, when also seeing himself as being researched, "[...] becomes confused with him. It is as close to the community as a member of the group that is studying and participates in its normal activities". This was evident, in particular, in the application of AOE, with the researcher's constant interventions, thus implying the involvement of students.

Finally, we clarify that AOE, in this study, in addition to being a methodological principle, was also presented as a data capture procedure. With this, we observe its possibilities of mediation in the process of appropriating trigonometric concepts when considering the relationship between students and researcher, from a collective perspective, especially in moments of reflection, raising hypotheses and in the search for a solution to the proposed learning-triggering activities.

With regard to the analytical procedure, aligned with the theoretical-methodological perspective embraced, we start from the Marxist maxim that objective reality exists independently of our consciousness, that is, it is not something given a *priori*. In fact, reality "[...] is not, therefore, a world of 'real', fixed objects, which under their fetishized aspect lead a transcendent existence as a naturalistically understood variant of Platonic ideas [...]" (KOSIK, 2011, p. 23, emphasis added, our translation).

In coherence with this analytical procedure, we rely on Moura (2004) when proposing the construction of learning episodes, understood as "[...] written or spoken phrases, gestures

and actions that constitute scenes that can reveal interdependence between the elements of a formative action" (MOURA, 2004, p. 276, our translation). These do not necessarily constitute a logical sequence of linear actions.

With this understanding, in the organization and selection of empirical data for this study, we identified the learning episode – Student involvement in the application of AOE –, the results of which we discuss in the following section.

Learning episode results and discussion

We dedicate this section to the results of the discussion produced through analysis and interpretation of the learning episode that emerged from the movement of applying AOE and the production of students' meanings about their potential in the process of conceptual appropriation.

Training meetings: the focus on AOE

This episode refers to the training scenario, consisting of four meetings, in which we contemplate the theoretical-methodological assumptions of AOE and, therefore, those of Activity Theory: logical-historical, need for the concept and interaction between students and mediation researchers and students. On that occasion, we applied two AOE. Specifically, about SDA, these were expressed based on students' everyday situations, entitled: *Calculating the height of large structures* and *finding the distance using trigonometric concepts* (BEZERRA, 2019).

Furthermore, in the training meetings, the SDA were shared by the collective, through the guidance of the researcher, since in the pedagogical activity, as Moura (2004, p. 257, our translation) explains, "[...] the teacher as a professional who deals with fundamentally with exchanges of meanings, he began to be seen as a learner who carries the elements that constitute his individuality constructed in a given culture".

Once this was done, the application of the first AOE began (Figure 4), in which the everyday situation had the triggering problem of discovering, based on trigonometric concepts, the height of a large chimney (Figure 3). To do this, we asked students how they could arrive at the mathematically correct answer to the problem.

Figure 3 – Chimney - the only remnant of the monumental União factory

Source: Bezerra (2019)

The aforementioned chimney, located on the premises of the research field school, now deactivated, was initially observed by the students, becoming part of the daily situation on screen. The conceptual need presented consisted of understanding the historical logic of trigonometric concepts. This, based on the objectification and appropriation of strategies in the search for answers to problems that trigger learning, was what made it possible to find the value of the distance and height of inaccessible places.

In this process, in the development of the study activity, the student "[...] makes himself by producing his objects and that, by producing his objects, he also produces his meanings whose maximum expression is the word" (MOURA, 2004, p. 260, our translation). In fact, since the beginning, among the problems experienced by humanity, as can be seen in the AOE *Calculation of the height of large structures* (Figure 4), when defining their housing and conditions for production, man was faced with different needs for control (LANNER DE MOURA, 1995).

Figure 4 – AOE – Calculation of the height of large structures

During a project carried out by Professor Joana on the history of the research field school, the curiosity of students Carlos, Débora and Wenderson was aroused. These students observed that, on their premises, there was a large chimney (Figure 5), which was abandoned at the back of the land and they were wondering how tall it was. They got together to try to find a way to do the calculations and then thought of some possibilities to find out the height, but were unsuccessful. Carlos thought about measuring using a tape measure, while Débora thought it was dangerous, due to the height. Débora had the idea of flying a kite and getting it stuck at the top of the chimney, to then measure the length of the line used. However, the use of this procedure is not permitted on school premises. Wenderson, quite curious, turned to the internet to research how to measure the height of large structures and, as a result, had his first contact with trigonometric concepts. However, he was unable to interpret these concepts. With the result of this research, the three students decided to do a study on how to calculate great heights and, thus, several questions arose, among which, it was configured as a problem that triggered learning: How can we use trigonometric concepts to discover height?

Source: Bezerra (2019)

It is evident in the triggering situation under analysis that, faced with the need to respond to the problem that catalyzes learning, namely: how to use trigonometric concepts to discover the height of the chimney, initially, the characters Carlos, Débora and Wenderson, possibly, in the first attempts, resorted to greatness of a discreet nature. However, they realized that when it comes to large values, counting one by one becomes insufficient and, in general, laborious. In this context, there are signs of elements of an imaginative and creative nature that enabled the development of his psyche and, by extension, different levels of meaning.

Thus, considering that in the proposition of this first AOE the actions are aimed at meeting the need to calculate the height of large structures, Carlos suggests that a tape measure be used for measurement. However, this possibility is ruled out by Débora as she observes the danger of an accident due to the height of the chimney. It is also worth highlighting that, in the measurement process, although the students adopted the measuring tape roll as a measuring instrument, just one roll would not be enough, when again they would be faced with the discrete magnitude and, consequently, this would constitute an obstacle.

Meanwhile, the character Débora proposed that they use a kite, attaching it to the top of the chimney. Thus, with a record of the measurement of the length of the line used, they would arrive at the height corresponding to the chimney. However, the rules established by the school's internal regulations hindered this strategy. Therefore, when new possibilities were raised, the character Wenderson, driven by curiosity to know how to measure the height of the chimney, resorted to consulting internet sites. In this action, he came across trigonometric concepts and, at the time, as he had not yet appropriated them, doubts arose that he was unable to interpret.

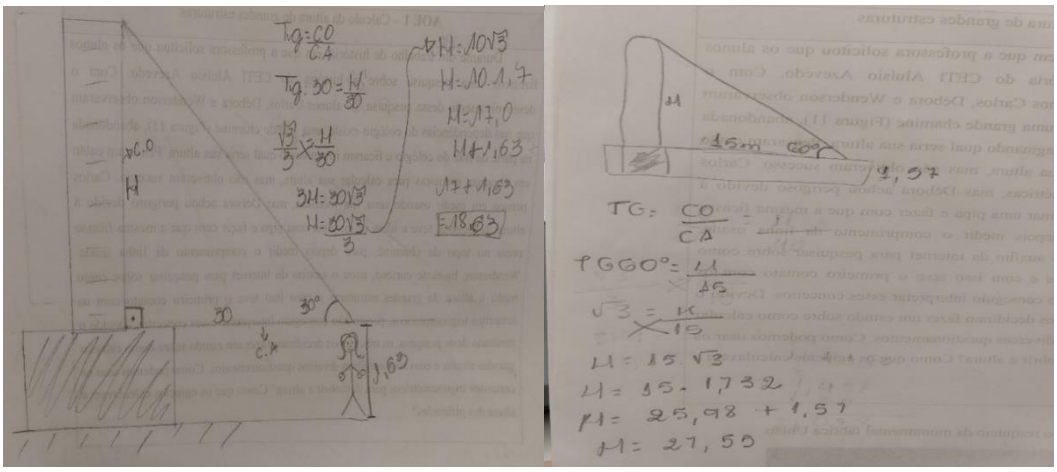
Still on the analysis of this SDA, given the strategies explained by the students in the search for answers to the problem that triggered learning, they had their reasons educated regarding the need to calculate the height of the chimney. Subsequently, to develop the calculations, the researcher began to problematize trigonometric concepts, with emphasis on controlling variations in the height of objects, which presented itself as a possibility for students to culturally signify measuring actions. In this regard, as said by Moura (2016, p. 116), in this way of organizing mathematics teaching, SDA has as its central objective "[...] providing the need for appropriation of the concept by the student, so that his actions are carried out in search of a solution to a problem that mobilizes him for the learning activity – the appropriation of knowledge".

From this perspective, the first actions developed, as described in the SDA by Carlos, Débora and Wenderson, were essential for the development of strategies that, under the intervention of the teacher/researcher, allowed the students to observe the variation in the elevation of the Sun. This had implications for the change in angle as a result of the sun's rays forming, together with the stick placed horizontally, a change in the size of this shadow. In this movement of thought, it was possible to think about the concept of tangent.

By way of illustration, when raising possibilities for resolving the triggering problem, Student 7 and Student 12 followed the same logic of thought, as analyzed by Bezerra (2019, p. 59, our translation) and described in figure 5.

> They thought about the hypothesis of its distance, making a simulation, to the chimney as being one of the sides of a right-angled triangle (adjacent side) and the height of that chimney (opposite side). In this way they reached the angle, looking at the top of the chimney. Here the students used the fundamental angles (30º, 45º and 60º), already appropriated by them. Therefore, as a final result of the calculation, they added the height of the observer to discover a possible answer. To do this, they had to adopt different angles and distances. In this way, based on the application of the concept of tangent in the right triangle, the students found the answers [...].

Figure 5 – Responses, respectively, from Student 7 and Student 12



Source: Bezerra (2019)

From the strategies used by the students in question, it was possible to observe divergences in the answers. Faced with this finding, the teacher/researcher, using problematization and intervention, led them to identify that, although the procedures had been used appropriately, the values corresponding to the distance and angle measurements were not correct, since to determine such measures, necessarily, should have auxiliary resources, such as the tape measure and the theodolite.

In this movement of conceptual appropriation, like trigonometric concepts, "[...] the way of approaching the concept also endows the subject with a new quality, when having to solve problems, because, in addition to having learned content new, it also acquired a way of appropriating content in general" (MOURA, 2016, p. 118, our translation).

Regarding the application of the second AOE – Discovering the distance using trigonometric concepts – we also used a situation experienced daily by students, as explained in figure 6.

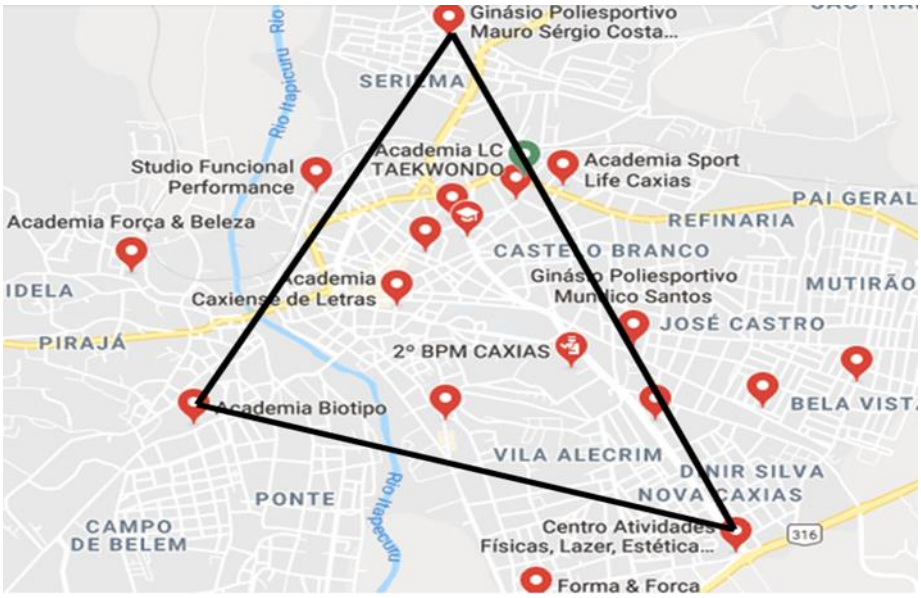
Figure 6 – AOE 2 – Discovering the distance using trigonometric concepts

The city of Caxias – MA was chosen by the Department of Education of the State of Maranhão to host the school games. As the venue for the games, the Caxias futsal team is already automatically classified. The games will take place between June 3rd and July 3rd, 2019. The opening match will take place at the Ginásio Poliesportivo Mauro Sérgio Costa, located in the Seriema neighborhood, while the final match will be held at the Physical Activities and Leisure Center, located in the neighborhood. New Caxias. Luciana lives in the Ponte neighborhood and plans to ride her bike to see the opening match, watch another game at Academia Biotipo, located in the same neighborhood, and go to the final match. As she often cycles around the city, she knows the distance between some neighborhoods. Between Ponte and Nova Caxias, it is 2100 meters; between Nova Caxias and Seriema it is 500 meters. Faced with this situation, we raise the triggering problem: What is the distance between the Ponte and Seriema neighborhoods, considering that the angle formed by the Ponte – Nova Caxias – Seriema neighborhoods is 30º?

Source: Bezerra (2019)

As seen, this time, the problem that triggers learning based on trigonometric concepts is the distance that covers the three spaces in the city of Caxias where the school games in the state of Maranhão were held. For better visualization, we present figure 7.

Figure 7 – Distance between gaming locations



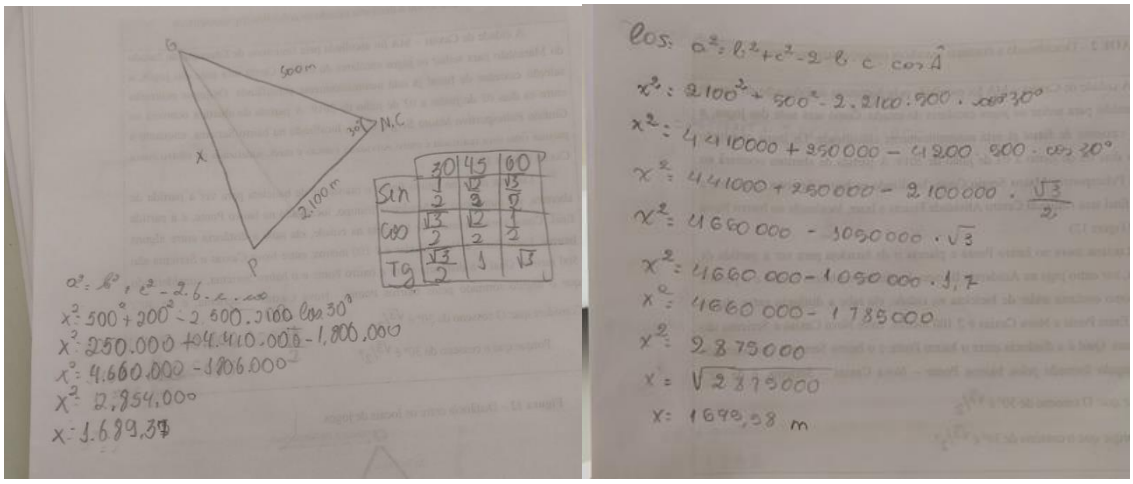
Source: Bezerra (2019)

In the development of the AOE under analysis, based on the teacher/researcher's interventions, the students presented some interpretations when proposing strategies in order to find answers that would meet the triggering problem. However, even with the study of the historical logic of trigonometric concepts and the interventions made, it was possible to observe,

on the part of some students, the attempt to solve the problem limited to trigonometric definitions in the right triangle. However, in this context of discussion, Student 8 and Student 10, upon recognizing that the triangle obtained from the distances between the three spaces where the games were played was not a right triangle, felt the need to expand their knowledge, reaching trigonometric concepts in any triangles and, by extension, the laws of cosines and sines.

Thus, collectively, students 8 and 10 came to the understanding that to respond to the problem that triggered learning, they would necessarily have to resort to the law of cosines, as shown in figure 8.

Figure 8 – Responses from Students 8 and 10



Source: Bezerra (2019)

In summary, regarding the answers presented by Students 8 and 10, despite presenting convergent strategies, they obtained different, although approximate, results. Certainly, this was due to some mistake made by the students when carrying out the calculations.

Given this situation, as we corroborate the theoretical reflections of Fraga *et al.* (2012), when discussing the concept of AOE and, also, as Moretti (2014, p. 33, our translation) advises, when emphasizing that "[...] creating learning conditions for subjects involves proposing problem situations to them that put them in front of the need for the concept", we opened space for conceptual problematizations, thus directing students to obtain answers that would coincide with those that, historically and socially, man determined as correct.

Having made considerations about the two AOE analyzed, it became evident the need for a first approach, on the part of the students, from the perspective of the historical logic of trigonometric concepts. To this end, learning conditions were intentionally created by the

teacher/researcher, thus constituting "[...] the situation actually proposed as a problem – and, therefore, as a necessity – for the subject who learns, making it possible for him appropriate the product of the historical construction of human culture" (MORETTI, 2014, p. 34, our translation).

In this direction, as complemented by Bezerra (2019) and Moura *et al.* (2010), in the search for problem solving, man develops abstractions that enable him to leave the chaotic level – of appearance (concrete starting point), to reach the thought level – of essence (concrete arrival point), thus ensuring, the appropriation of humanity's historical-social experience.

Thus, we clarify that this movement of thought was what led us, from the discussion on the similarity of triangles, to problematize and demonstrate the trigonometric relations in the right triangle. To do this, we use the tape measure and the theodolite as complementary mediating instruments in the development of AOE, in the search for resolving the triggering problems.

Based on the above, the need to create, in school environments, moments in which the subjects involved can interact and negotiate meanings is evident, configuring a condition for reflection on the pedagogical activity and, logically, the organization of teaching.

From the above, we can state that the meanings developed by students about the potential of AOE are in line with what Moura *et. al.* (2016, p. 99, our translation), "in order to form the student's theoretical thinking, it is necessary to organize teaching so that they carry out activities suitable for the formation of this thinking". In summary, AOE constitutes a possibility of overcoming the level of chaotic concrete thinking to the concrete thought, resulting in the development of thinking at a theoretical level.

Finally, the reflections on the meanings developed by the students express the need to resize the actions of the pedagogical activity. From this perspective, this is one of the characteristics of AOE, in addition to reflection as an enabler of a method for appropriating knowledge (RODRIGUES; SFORNI, 2010; SFORNI, 2004).

Final remarks

The time has come to summarize our reflections on the possibilities of AOE as a methodological principle in the process of appropriating trigonometric concepts. To do this, we make an analogy to the thought of Araújo and Moura (2008), cited by Bezerra (2019, p. 74, our translation), when expressing the completeness of the teacher's identity process: "[...] a master is the one who suddenly learns, the one who learns a lot, because it's almost always sudden." In our opinion, it coincides with the movement carried out in this study, in which we suddenly learned... we learned a lot.

Given the need to organize teaching, we sought answers to the question of this study, namely: how does AOE, as a methodological principle, enable mediation in the process of appropriating trigonometric concepts? To this end, as we understand that "a pedagogy that aims to be revolutionary must, therefore, start from the history itself and the productions of human beings in the educational field" (MARQUES; DUARTE, 2020, p. 2209, our translation), initially, by proposing to students the study of the historical logic of mathematical concepts, we created the first conditions for their involvement in the application of AOE.

In general terms, this study involved raising awareness, both on the part of the students, research participants, and the researcher that, when organizing Mathematics teaching, it is necessary to consider the historical logical movement of the concept understood as a human need (KOPNIN, 1978). We understand, in this logic, that this allows, above all, the teacher, to appropriate theoretical-scientific knowledge from the perspective of mediation, when considering AOE, a maxim defended by Moura *et al.* (2010).

Given the evidence revealing the possibilities of participants, in a situation of experiencing and developing AOE, to appropriate trigonometric concepts, it is necessary to consider, among other factors: the central mathematical concept involved; does the SDA present questions that enable the essence of the concept?; Is the context worked on, for example, based on emergent everyday situations, under the mediation of guided games?; Does the SDA text provide clarity so that the student recognizes the triggering problem?

In this sense, in other words, we understand that, when the teacher adopts AOE as a methodological principle, with emphasis on the triggering problem, this must necessarily be linked to his/her teaching level. To this end, the starting point must be the historical logical movement of mathematical concepts, as this will have implications in the sense that students' motives are educated in the face of the need to find a solution to the triggering problem. From this perspective, it is important to emphasize that only when the reason coincides with the

objective of the SDA can we point out that the student is engaged in a learning activity. In the case of this study, this possibility was realized through the elaboration and development of AOE, which, certainly, implied in the organization of the teaching of trigonometric concepts. This assumption is in line with what Leontiev (1978) states about activity as a promoter of the subject's psychic development.

In view of the above, among the contributions of this study, there is undoubtedly the expansion of studies in the field of Mathematics Education, subsidized by the Activity Theory and the assumptions of AOE. On the other hand, it is a study with enriching potential for professional development (teaching and/or researcher). This led students and research professors to produce meanings about pedagogical activity, which go against perspectives based on technical rationality, limited to definitions and formulas, devoid of meanings. Therefore, the AOE proposal involving trigonometric concepts in high school can generate changes in thinking and actions in the face of concerns and problems within the scope of the pedagogical activity of teachers who teach mathematics. Here's what we expect from now on!

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