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LEARNING AND MATHEMATICAL THINKING: A LOOK AT PROBLEM-SOLVING AND POSING BY CHILDREN IN THE EARLY YEARS

APRENDIZAGEM E PENSAMENTO MATEMÁTICO: UM OLHAR A PARTIR DA RESOLUÇÃO E PROPOSIÇÃO DE PROBLEMAS POR CRIANÇAS DOS ANOS INICIAIS

APRENDIZAJE Y PENSAMIENTO MATEMÁTICO: UNA MIRADA DESDE LA RESOLUCIÓN Y PLANTEAMIENTO DE PROBLEMAS POR PARTE DE LOS NIÑOS EN LOS PRIMEROS AÑOS

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ABSTRACT: This study aims to analyze the association between problem-solving and posing and its potential to promote learning and aid in understanding students' mathematical thinking. To this end, a qualitative case study was carried out involving elementary Grades 1-5' students in the solving of a problem that was adapted to generate different learning outcomes related to the division operation as well as associating the posing of problems with the request to add a question to the problem. The results confirm the importance of teachers' fostering of discussions and understanding students' mathematical thinking; they also reveal that difficulty in interpreting the problem statement may not always be the reason for students not providing a solution. In addition, the data shows the potential of problem posing to deepen or advance the learning that results from problem-solving.

KEYWORDS: Problem-solving. Problem posing. Interpretation of the statement. Implemented curriculum. Teaching mathematics.

RESUMO: *Este estudo tem o objetivo de analisar a associação entre a resolução e a proposição de problemas e seu potencial para promover a aprendizagem e ajudar a compreender o pensamento matemático dos estudantes. Para isso, foi realizada uma pesquisa qualitativa do tipo estudo de caso, envolvendo estudantes do 1º ao 5º ano do Ensino Fundamental na resolução de um problema que foi adaptado de modo a gerar diferentes resultados de aprendizagem em relação à operação de divisão, bem como associa-se à proposição de problemas a partir da solicitação de adicionar uma pergunta ao problema. Os resultados confirmam a importância de o professor fomentar discussões e compreender o pensamento matemático dos estudantes; eles também revelam que a dificuldade em interpretar o enunciado do problema pode, nem sempre, ser a razão pela qual os alunos não apresentam uma solução. Adicionalmente, os dados relevam o potencial da proposição de problemas para aprofundar ou avançar nas aprendizagens decorrentes da resolução de problemas.*

PALAVRAS-CHAVE: *Resolução de Problemas. Proposição de Problemas. Interpretação do enunciado. Currículo implementado. Ensino de Matemática.*

RESUMEN: *Este estudio tiene como objetivo analizar la asociación entre la resolución de problemas y la propuesta de problemas y su potencial para promover el aprendizaje y ayudar a comprender el pensamiento matemático de los estudiantes. Para ello, se realizó una investigación cualitativa del tipo estudio de caso, involucrando a estudiantes de 1° a 5° año de Educación Básica en la resolución de un problema que se adaptó con el fin de generar diferentes resultados de aprendizaje en relación con la operación de división, así como asociado a la propuesta de problemas a partir de la solicitud de agregar una pregunta al problema. Los resultados confirman la importancia de que el docente fomente las discusiones y comprenda el pensamiento matemático de los estudiantes; También revelan que la dificultad para interpretar el planteamiento del problema no siempre es la razón por la que los estudiantes no encuentran una solución. Además, los datos revelan el potencial de la propuesta de problemas para profundizar o avanzar en el aprendizaje resultante de la resolución de problemas.*

PALABRAS CLAVE: *Resolución de problemas. Proposición de problemas. Interpretación de la declaración. Currículo implementado. Enseñanza de las Matemáticas.*

Introduction

The curricula of several countries highlight problem-solving as an important part of what it means to teach and learn mathematics (Brasil, 2018; National Council of Teachers of Mathematics, 2000), and, indeed, problem-solving is the heart of mathematics (Polya, 1985; Cai; Hwang, 2021). However, despite abundant research, implementing problem-solving as a means of teaching mathematics is still a challenge for teachers due to beliefs about what it means to learn mathematics or what problem-solving is.

Teaching through problem-solving is a different environment from a lesson that begins with the teacher presenting the content and some examples, and then presents problems for which the solving processes are already known or are prescribed by the teacher. When teaching to solve problems, the student's role is limited to applying the content, and repeating what has been taught. In teaching through problem-solving, students are not conditioned by example but mobilize their prior knowledge and make connections, presenting solutions or obtaining solutions that sometimes surprise the teacher and may include unforeseen strategies. This may also be one of the reasons why teachers are hesitant about teaching through problem-solving (Liljedahl; Cai, 2021).

Like problem-solving, problem posing has been of great interest in mathematics education research. More recently, it has also been suggested that the problems to be solved should be posed not only by the teachers and especially by the students (National Council of Teachers of Mathematics, 2020). Both problem-solving and problem posing are important resources for moving from the prescribed curriculum to classroom implementation. However, despite extensive research on teaching through problem-solving and problem posing, the relationship between these two strands is still not evident in curriculum materials (Cai; Hwang, 2021; Possamai; Allevato; Strelow, 2023).

Problem-solving activities, including those in textbooks, can be used as opportunities to integrate problem posing into mathematics classes. For example, teachers can remove some information from an original problem and ask students to complete or supplement it with new data or information (Cai; Hwang, 2021) or they can ask the students to create or modify the original problem so that it becomes more difficult (Allevato; Possamai, 2022; Cai *et al*., 2023).

The research presented in this article aims to analyze the association between problem-solving and posing and its potential to promote learning and aid in understanding students' mathematical thinking. Some of the theoretical aspects that guide this study are presented below, followed by the results and discussion.

Problem-solving and Posing

Although the importance of problem-solving in mathematics teaching is unquestionable, teachers do not always have a clear position on what a problem is or what problem-solving looks like, impacting the way it is developed in the classroom (Allevato, 2014; Bailey, 2022; Lester; Cai, 2016).

Problem-solving is explored in this study with the understanding that a problem "is a task presented to students in an instructional setting that poses a question to be answered but for which the students do not have a readily available procedure or strategy for answering it" (Lester; Cai, 2016, p. 122, our translation) and which, therefore, cannot be solved by procedural effort alone. This understanding is important because it is reflected in the way problem-solving is implemented in the classroom, given how often teachers include problems and problem-solving only as a "complement" that gives students the opportunity to apply or practice a specific procedure previously taught (Bailey, 2022; Allevato, 2014).

When teaching through problem-solving, learning takes place during the process of searching for the solution to the problem, throughout which relevant mathematical concepts and skills are incorporated (Allevato, 2014; Lester; Cai, 2016) and in an instructional environment that makes it possible to learn mathematics through social interactions, negotiating and producing meaning, and developing increasingly connected systems of knowledge (Allevato, 2014; Allevato; Onuchic, 2021; Bailey, 2022).

From this perspective, the problem is considered the starting point and provides orientation for learning new mathematical concepts and procedures (Allevato; Onuchic, 2021). However, it should be emphasized that it is not the format of the problem that defines its potential but rather the actions of the teacher, conditioning the results of the discourse (or lack thereof) in the classroom of the problem-solving processes used. It is important to reiterate that

> the most important criterion of a worthwhile mathematical problem is that the problem should serve as a means for students to learn important mathematics. Such a problem does not have to be complicated or have a fancy format. As long as a problem fosters students' learning of important mathematics, it is a worthwhile problem (Lester; Cai, 2016, p. 123, our translation).

But where do these problems come from? In this vein, Getzels (1979) distinguished between three types of problems: those presented, those discovered, and those created. In mathematics classes, students are generally asked to solve problems that are presented to them and it is up to them to solve them and the teachers to create them or to select or adapt them from textbooks or other materials.

However, encouraging students also pose problems is present in current curriculum documents and guidelines and has been the focus of research in mathematics education as it is perceived to be more cognitively demanding and, at the same time, more accessible than problem-solving (Cai, 2022; Possamai; Allevato, 2023). In addition, “problem posing as a goal of mathematics instruction is linked to the promotion of mathematical activities or of positive affect (mathematical curiosity, interest, enjoyment) that encourages students and teachers to pose new and better problems” (Cai; Leikin, 2020, p. 297, our translation).

It should be emphasized that problem posing refers to an activity in which students are responsible for creating a problem and presenting it to a potential solver, as opposed to solving a given problem, usually presented by the teacher, which is how problem-solving usually takes place in the classroom. In this way, while problem-solving involves learning during the process of searching for a solution, in teaching through problem posing, learning takes place during the process of creating and discussing the problems created; let's repeat, created by the students. However, even though they are different activities, they complement each other and can thus enhance mathematical learning, since by analyzing the problems posed by the students, it is possible to identify conceptual learning gaps that need to be worked on (Possamai; Allevato, 2023) and thus make them better problem solvers.

There are different possibilities for integrating problem-solving activities into the classroom, and one simple strategy involves associating them with problem-solving by making small changes or insertions to the problems available in textbooks. Recent research (Cai, 2022; Cai; Hwang, 2021) has shown that these strategies, however modest, can encourage and significantly increase teachers’ ability to include problem-solving in their lessons.

This study analyzes how students pose problems by adding a question to a problem that has already been solved and discussed in class, as shown below.

Methodological Characterization

The study presented in this article is qualitative, aiming to attribute meaning to the phenomena investigated (Amando; Freire, 2014) and specifically to analyze the association between problem-solving and posing and its potential to promote learning and aid in understanding students’ mathematical thinking.

The research involved 14 Grades 1−5 students at a public elementary school in Santa Catarina, Brazil. The students were chosen at random by a teacher from the school who helped the researchers collect the data. The students were asked to solve a generator problem and then to pose a new question for the problem, with the aim of developing learning related to division and moving on to posing problems. The generator problem was adapted to the students' school level, and for the first- and second-grade students it was implemented using the activity shown in Figure 1.

Figure 1 – Problem-solving and problem-posing activities used by Grades 1 and 2 students

Ricardo has a 38-page book. He has already read 24 pages of the book and wants to finish it in two days, reading the same number of pages each day. a) Mark an X in the following questions that can be answered and present the solving. ( ) How many days did it take him to read the 24 pages? ( ) How many pages does he have left to read? ( ) What is the name of the book? ( ) How many pages should he read a day? b) Create a new question that could be answered.

Source: Adapted from Itacarambi (2010)

Solving the problem required the students to build strategies for doing division, particularly for the idea of half, which had not yet been discussed in class. This problem was adapted for the students in higher grade levels, changing what was announced, as shown in Table 1, and consequently the data in item (a).

Table 1 – Adaptations of the problem for higher grade levels

Grade 3	Ricardo has a 94-page book. He has already read 28 pages of this book and wants to finish it in two days, reading the same number of pages each day.
Grade 4	Ricardo has a book of 340 pages. He has already read 264 pages of this book and wants to finish it in 4 days, reading the same number of pages each day.
Grade 5	Ricardo has a 340-page book. He has already read 264 pages of this book and wants to finish it in 5 days.

Source: Prepared by the authors

The generating problem for third grade involves higher order subtraction and the division involves larger numbers. The problem for fourth grade involves division by 4, going beyond the idea of half. Finally, the fifth-grade problem involves division with a remainder or decimal result.

Item (b) was intended to show how the posing of problems is associated with problem-solving, in these cases as a possibility to deepen the learning resulting from the problem already solved or to move on to new learning.

The research data initially consisted of the students' written records, which were collected by the collaborating teacher. After analyzing the students' solutions to the problems posed, the researchers conducted interviews with some of the students to gain a better understanding of the processes involved in their decisions and solutions. The data is presented with the students identified by the "student-year" code; for example, 1-5 refers to Student 1 in Grade 5.

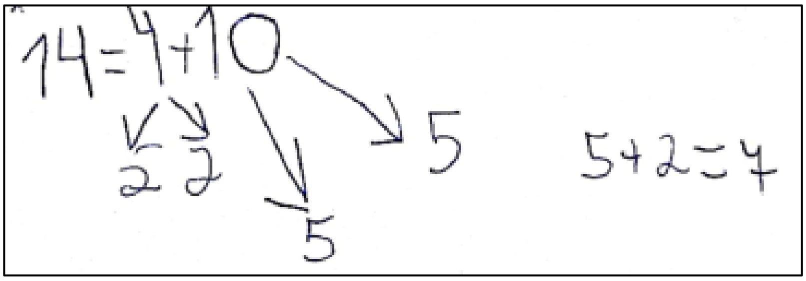
This investigation is thus a case study because when it was conducted with this group of students, the intention was not to produce generalizations with the report of the results or, in other words, to suggest that others could match the emerging data but rather to provide indications of relevant aspects to be considered in other investigations and in classroom educational practices.

Regarding methodological aspects of the study, in the next section the data are presented and analyzed.

Data Reporting and Analysis

The activity in Figure 1 was carried out by two first graders and one second grader. All three children were able to determine how many pages were left to finish reading the book and carried out subtraction. However, only one of the children was able to identify that she could answer how many pages should be read each day to finish reading the book and came up with a solution. The solution is shown in Figure 2.

Figure 2 – Grade 2 student's solution (Student 1-2)



Source: Research collection

It is important to reiterate that the division operation had not yet been covered in class. However, this student managed to answer the question using strategies related to decomposing numbers in a way that made sense to her. When asked about her solution, the child explained: "I know that 4 is 2 + 2 and 10 is 5 + 5; so, I know what half of each is, and then I put the halves together, 5 + 2 is 7." This emphasizes the importance of problem-solving as an "active and constructive process in which students carry out classroom activities in the light of their beliefs and assimilate information within their pre-existing knowledge structures. As a result, each student builds a 'personalized' type of mathematics" (Allevato; Onuchic, 2019, p. 2, our translation).

The other two children didn't identify that the question "how many pages should be read a day" could be answered, and they were also interviewed. The researcher asked, reducing the number: "And if there were only 10 pages left, would it be possible to know how many should be read each day?" Even so, the children were unable to come up with a strategy. The researcher continued: "If I read one page today and another tomorrow, would I finish the book?" The children answered no but were unable to move forward and establish a strategy to solve the question and determine how many pages should be read each day. Possibly, these students still lacked experiences developing number sense. This would have enabled them to think flexibly about numbers and make progress in building strategies involving division.

All three Grade 3 students were able to determine how many pages were left to finish reading by structuring the standard subtraction algorithm. However, none of the students answered the question "How many pages should he read a day?" Two of them indicated that the question could be answered but, unable to come up with a solution, erased what they had marked. After analyzing these records, the researchers first interviewed the teacher of the class to which these third graders belonged:

Researcher: We've noticed here that the students marked that they could answer the question, but erased it. Knowing the students and the classroom context, why do you think they marked that they could solve it, but didn't?
Class teacher: I think they were unsure when they read it. That's why when I do an assessment with them, I make the questions as simple as possible and I tell them that the answer is in the text and ask them to read it carefully. Sometimes they do it in a hurry and get nervous, and end up not answering as they should. Their problem is interpreting the wording.
Researcher: Had they already learned about division when they solved this problem?
Class teacher: They hadn't learned. But they find it very difficult to interpret problems.

This teacher's account corroborates what has already been found in other research (Lester; Cai, 2016), that teachers often eliminate the challenges of a mathematical task by showing students how to solve the problem or by reducing the level of difficulty by leading them to the solution. This stems from an education in which teachers have been taught to "help their students" by reducing the level of difficulty of problem-solving for students, which is based on the traditional conception of teaching.

Regarding this, Polya (1985, p. 15, our translation) advises that:

> If we regard the development of the pupil's intelligence as the main (or one of the most important) aims of teaching at secondary level, and the pupil's work on solving problems as the main (or one of the most important) means of achieving that end, then the teacher's main (or one important) concern should be to lead the pupil to discover the solution for himself.

It should also be noted that, naively, the teacher persistently attributes the students' difficulty to their "difficulty in interpreting" the problem/statement, even neglecting the fact specifically linked to mathematics - that those students had not yet worked with division in class.

The researchers then interviewed the students. First, the students were asked to read the problem again and then the researcher asked:

Researcher: I saw here that you marked that you could answer and deleted it. How many pages can he read a day? Why did you delete it? Do you remember?
Student 1-3: It's just that I didn't know how to solve it. I couldn't solve it and then I erased it.
Researcher: But can you see that it's possible to solve with the data in the problem? [Student nods yes] Would you be able to answer if you didn't have to do the math?
Student 1-3: I think that to read in two days he has to read 33 a day.
Researcher: Why do you think that?
Student 1-3: Because half of 60 is 30 and half of 6 is 3. And 33 + 33 makes 66.
Researcher: You knew that, but you didn't answer because you don't know how to do division yet [Student nods in agreement].

Another student (2-3), who also marked that the question could be answered, when interviewed had no idea how to answer the question. He tried to find some operation in his knowledge that would fit the demand, answering that he should do 66 - 2.

These data indicate that, being conditioned to present the standard account (algorithm) to the teacher, the students felt unable to solve the problem because they did not know the algorithm. This type of result is consistently documented by several researchers when they verify, as indicated by Lester and Cai (2016), that "students' beliefs about the nature of problem-solving are not restricted to how problems should be solved. Many students also have firm beliefs about what is expected of them when their teachers give them problems to solve" (p. 126, our translation).

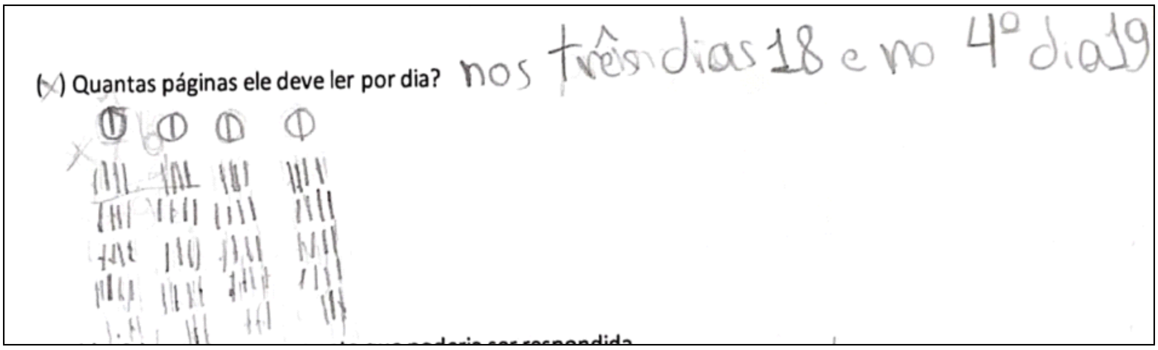
We highlight two solutions developed among the three fourth-grade students. One of the students structured the division algorithm (Student 3-4), and the other used his knowledge of numbers and the relationships between operations to obtain the solution, confirming it with the counters register, as shown in Figure 3.

Figure 3 – Student 1-4's solution

Source: Research collection

Another student used only registration using counters being drawn, as shown in Figure 4.

Figure 4 – Solving the problem with counters, Student 2-4



Source: Research collection

Looking at Figure 4, we can see that the student correctly distributes 19 counters in each column but miscounts when presenting the solution. These fourth-grade students were more flexible in their recording, which possibly reflects the teacher's approach and mediation in the classroom, allowing them to solve problems using strategies they considered coherent and meaningful to them.

Of the fifth-grade students, two did not identify that it would be possible to determine how many pages would be read per day (Students 4-5 and 5-5). The other three structured the division algorithm, obtaining the following answers: "It must be 15.2 pages to finish the book" (Student 1-5); "15 pages a day" (Student 2-5; this student obtained a remainder of 1 and ignored it); and one of them misinterpreted the question to divide 264 by 5 (Student 3-5).

The student who answered 15.2 was interviewed:

Researcher:	What did you mean by 15.2 in your answer? What does it mean?
Student 1-5:	As I had some left over, I asked the teacher if I could put in a comma, so I did. Then I thought that this "comma 2" would be half a page, so he would have to read 15 and a half pages every day.

The student who answered with a remainder of 1 was also interviewed:

Researcher:	What does that 1 down there mean?
Student 2-5:	It's just that there are leftovers.
Researcher:	Could you continue? What do you think it means?
Student 2-5:	It means 1 page.
Researcher:	And what do we do with it?
Student 2-5:	Do I have to put in a comma?
Researcher:	If I don't put the comma, what do I do with that 1?
Student 2-5	He would have to read part of the page a day more. [The student picks up the paper and continues using the comma and arrives at 15.2]
Researcher:	So what? If I took the book to read, I would read 15 pages and what would I do with "comma 2?"
Student 2-5:	I would look at the lines. I'd have to read 2 lines... or half of half.

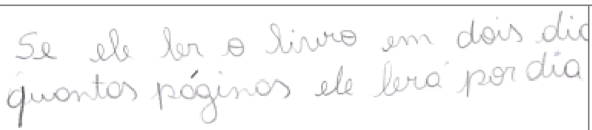
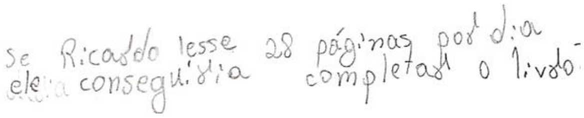
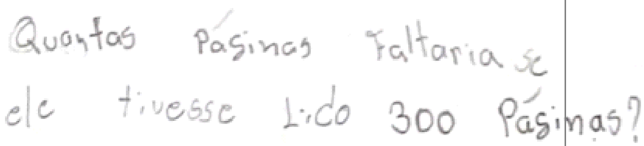
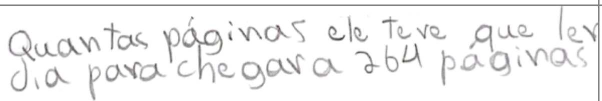
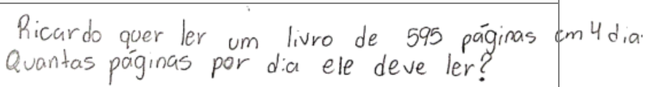
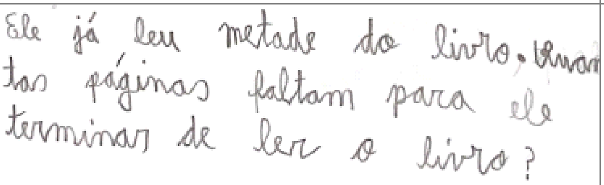
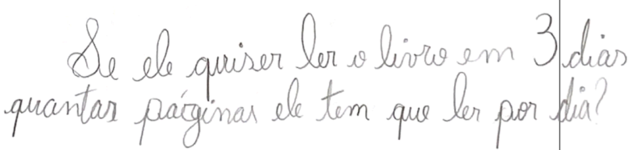
The student is unable to express his understanding of the rest, which would imply that one day he would read 16 and not 15 pages. When he moves on to the result, 15.2, he realizes that 0.2 is less than half the page, but he can't express exactly what it means. These interviews highlight the potential of this problem to advance students' learning of rational numbers in decimal form because although they know how to operate with division, there is no clear understanding of its meaning.

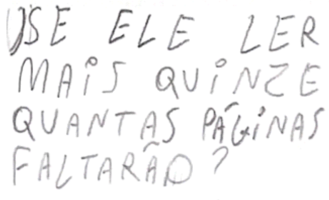
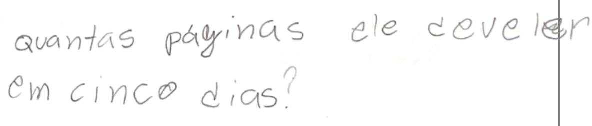
When the results are discussed and a consensus is reached on the best or correct solution, by the teacher together with the students, and even when the content is formalized by the teacher, these solutions enable the teacher to lead the students to establish relationships between rational numbers in decimal form, fractions, and percentages, producing meaning for the results found and advancing students' learning of rational numbers. This reconfirms problem-solving as "fruitful in building knowledge of the great ideas present in mathematics" (Allevato; Onuchic, 2019, p. 4, our translation), which makes it possible to establish important mathematical connections.

These data also reinforce the importance of the teacher in problem-solving which, according to Lester and Cai (2016), can have two objectives: (a) to solve the problem and (b) to encourage students to reason and reflect on their reasoning, demonstrating an understanding of the mathematical concepts embedded in the problem. When the teacher's objective is simply for the students to solve the problem, they accept solutions that consist of sequences of calculations. However, in the second perspective, the teacher is more persistent in probing the students' thinking and the meaning attributed to the calculations presented (Lester; Cai, 2016).

As for creating a question that could be answered, this was a demand that the students found strange, and one first grader, two third graders, and one fourth grader left this item blank. Table 2 shows the questions created in response to item (b) of the problem.

Table 2 – Questions posed by the students

 If he reads the book in just two days, how many pages will he read each day? Grade 1 (transcribed by the teacher) - Student 1-1	 How many pages could he read in 3 days? Grade 2 - Student 1-2
 If Ricardo read 28 pages a day, would he be able to complete the book? Grade 3 - Student 1-3	 How many pages would be missing if he had read 300 pages? Grade 4 - Student 3-4
 How many pages did he have to read a day to reach 264 pages read? Grade 4 - Student 2-4	 Ricardo wants to read a 595-page book in 4 days. How many pages a day should he read? Grade 5 - Student 1-5
 He's already read half the book. How many pages are left before he finishes reading the book? Grade 5 - Student 2-5	 If he wants to read the book in 3 days, how many pages does he have to read each day? Grade 5 - Student 3-5

	
If he reads fifteen more, how many pages will be left?	How many pages should he read in five days?
Grade 5 - Student 4-5	Grade 5 - Student 5-5

Source: Research collection

The question posed by the first grader, “If he reads the book in just two days, how many pages will he read each day?” is an interesting modification of the original problem and involves dividing the 28 pages into 2 days. The question shows that the student, despite not being able to come up with a strategy to carry out the division, was able to see that it was a question that could be addressed by the problem. In other words, the question had become a problem for the posing student himself, since he didn’t know how to answer the original problem of dividing 14 pages into 2 days.

This reveals the potential of problem posing to advance and deepen the learning and understandings produced by problem-solving, given that

> both experiences with problem-solving help students to understand the structure of a problem in order to then create their own problems, and the posing of problems improves the processes of reading, interpreting and solving problems, since students need to analyze the structure and objective of the problem in their creation process (Possamai; Allevato; Strelow, 2023, p. 151, our translation).

In addition, it is important to emphasize the importance of posing problems so that students are involved in the personal interpretation of the problem, expressing their own mathematical thinking, and also stimulating critical and creative thinking (Cai, 2022; Possamai; Allevato, 2024; Vieira; Possamai; Allevato, 2023).

The second grader asked “How many pages would he read if it took 3 days?” which was also asked by a fifth grader. This is also an interesting problem because it involves division with a remainder, making it possible to move on to new learning. It should be emphasized that although the fifth-grade student’s repertoire of mathematical knowledge and content is supposedly greater than that of the second-grade student, both students generated similar problems. This reiterates the idea that

> although problem-posing activities are cognitively demanding tasks, they are adaptable to students’ abilities and thus can increase students’ access such that

> students with different levels of understanding can still participate and pose potentially productive problems based on their own sensemaking (Cai, 2022, p. 32, our translation).

The posed questions involved changing the number of pages, days, or pages read per day while remaining linked to the original problem. Only one student, from fifth grade, posed a new problem: "Ricardo wants to read a book of 595 pages in 4 days. How many pages a day should he read?" which as an extension of the original problem.

Different strategies can be used by the teacher to promote the discussion of these problems posed in class, such as asking the children to swap the questions between them; compiling all the questions into a list and asking the children to choose the two they find most interesting to solve; or asking the students to choose one they find easy and one they find difficult to solve. The important thing is to recognize the potential of these questions to assess, deepen, or broaden the learning derived from the problem initially solved (Cai, 2022; Possamai; Allevato, 2023).

Finally, we conclude that this activity is a rich and truly challenging situation for the students, allowing them to analyze which questions make sense and to generate and explore mathematical ideas while also being able to produce their own questions by posing problems. It is also clear that the teacher's approach has the power to enhance the results (Lester; Cai, 2016), specifically in how they promote discussion and how they enable and encourage students to look for different strategies to solve the original problem and the new problems created from the posed questions.

Final remarks

The aim of this study was to analyze how associating problem-solving with problem posing can promote learning and better understanding of students' mathematical thinking, based on a case study. The data show the importance of carrying out research of this type to advance our understanding and comprehension of students' mathematical solutions, highlighting important issues to be considered when working in the classroom.

The results highlight the importance of the teacher's role in encouraging discussions and carrying out interventions that enable students to express their understanding of the mathematics involved in a problem, valuing their solutions and mathematical reasoning. When the classroom is restricted to analyzing answers and calculations, without exploring the resulting meanings, the potential for establishing connections and advancing mathematical learning is reduced.

The data also show that attributing students' lack of a solution to a problem to difficulties in interpreting the problem is a naive conclusion as to what might actually be occurring with the students, and this is a recurring theme in the teachers' discourse. As we have seen, students may also not come up with a solution because they believe that the way they can solve the problem differs from what they believe the teacher expects—in particular, that they are limited to the use of algorithms. In addition, they may not have a minimal conceptual grasp of central aspects of the mathematics involved in the problem.

This reinforces the idea that it is not the format or sophistication of the problem that guarantees good results in terms of student learning but rather the time and discourse that is allowed (or not) to take place in the classroom, encouraging and enabling students to share how they think and construct meaning in mathematics.

This study demonstrates the potential of problem posing even when incorporated as a simple demand, such as asking for an additional question to be posed for a problem. It should be emphasized that despite the call from research and curriculum documents to integrate problem posing into school mathematics, there has been little progress in terms of its presence in textbooks let alone in the curriculum implemented in classrooms.

The results of this study show that problem posing presents a way to deepen or advance the learning that comes from problem-solving, allowing students to create problems of different levels of complexity regardless of their level of mastery of mathematical knowledge. This data makes it possible for teachers to play a leading role in the implementation of the curriculum and for problem posing to truly take root in school mathematics, based on simple modifications to available materials.

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