"HOW DO TEACHERS LEARN?": PROPOSALS FOR THE DEVELOPMENT OF MATHEMATICAL KNOWLEDGE

"COMO OS PROFESSORES APRENDEM?": PROPOSTAS PARA O DESENVOLVIMENTO DO CONHECIMENTO MATEMÁTICO

"¿CÓMO APRENDEN LOS DOCENTES?": PROPUESTAS PARA EL DESARROLLO DEL CONOCIMIENTO MATEMÁTICO

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ABSTRACT: In this article we present partial results of two researches that proposed to investigate the possibilities of how the teacher learns Mathematics to teach and develop mathematical reasoning in students. Thus, the question that guided this article was: Which formative actions can contribute more effectively to the development of mathematical and/or didactic knowledge? The study is the result of a qualitative descriptive research and presents the results of two projects, one developed in the state of Rio Grande do Norte and the other developed in the state of Mato Grosso do Sul. In general, as a result, we present reflections and data that can contribute to the promotion of spaces for dialogue and construction of mathematical and pedagogical knowledge, aiming at the development of specialized knowledge for the teaching of mathematics and the articulation between theory and practice.

KEYWORDS: Teacher education. Mathematics Education. Neuroscience. Children's literature.

RESUMO: *Neste artigo são apresentados resultados parciais de duas pesquisas que se propuseram a investigar as possibilidades de como o professor aprende Matemática para ensinar e desenvolver o raciocínio matemático nos estudantes. Assim a questão que norteou esse artigo foi: Quais ações formativas podem contribuir de maneira mais efetiva para o desenvolvimento do conhecimento matemático e/ ou didático? O estudo é fruto de uma pesquisa qualitativa do tipo descritiva e apresenta os resultados de dois projetos, um desenvolvido no estado do Rio Grande do Norte e outro desenvolvido no estado do Mato Grosso do Sul. De modo geral como resultados apresentamos reflexões e dados que podem contribuir para a promoção de espaços de diálogo e construção de conhecimentos matemáticos e pedagógicos, visando o desenvolvimento do conhecimento especializado para o ensino de matemática e a articulação entre teoria e prática.*

PALAVRAS-CHAVE: *Formação de professores. Educação Matemática. Neurociência. Literatura infantil.*

RESUMEN: *Este artículo presenta resultados parciales de dos estudios que tuvieron como objetivo investigar las posibilidades de cómo los profesores aprenden Matemática para enseñar y desarrollar el razonamiento matemático en los estudiantes. Así, la pregunta que orientó este artículo fue: ¿Qué acciones formativas pueden contribuir de manera más efectiva al desarrollo del conocimiento matemático y/o didáctico? El estudio es el resultado de una investigación cualitativa descriptiva y presenta los resultados de dos proyectos, uno desarrollado en el estado de Rio Grande do Norte y otro desarrollado en el estado de Mato Grosso do Sul. En general, como resultado, se presentan reflexiones y datos que pueden contribuir a la promoción de espacios de diálogo y construcción de conocimiento matemático y pedagógico, con el objetivo de desarrollar conocimientos especializados para la enseñanza de la matemática y la articulación entre teoría y práctica.*

PALABRAS CLAVE: *Formación del profesorado. Educación Matemática. Neurociencia. Literatura infantil.*

Introduction

How do teachers learn to teach? This question may seem simple at first glance, but it is still a mystery for researchers in the area of Teacher Training (Coimbra, 2020; Gatti, 2021; Bogatschov; Ferreira; Moreira, 2022). Although teacher training has followed the evolution of public educational policies in recent decades, it still faces complex, profound and many challenges, such as: i) inequalities regarding the quality of courses; ii) lack of attractiveness; iii) discontinuation of studies and training; and iv) the devaluation of the profession. Although the improvement of educational quality is related to several factors in the school context Hanushek (2020), the low performance of students in macro assessments directly leads us to think about the teaching practice developed by teachers who teach Mathematics in the classroom and also in the how important the role of the teacher is so that the construction of knowledge is actually favored.

Underlying many of these concerns is the shortage of teachers prepared to teach mathematics, with the shortage being most pronounced in places where students appear to need the most assistance. Research such as Ball (1990), Attorps (2003), Barbosa (2009) and Lautenschlager; Ribeiro (2014), among others, indicate that many teachers of the subject do not have a conceptual understanding of many elementary Mathematics contents and, therefore, end up privileging in their classes the development of algorithmic skills, in a segmented and content-based way, without privileging mathematical thinking.

Another problem highlighted by several studies in Brazil (Fiorentini; Oliveira, 2013; Moreira; Ferreira, 2013) is the gap between the Mathematics taught in initial teacher training courses (Degrees) and the mathematical practices actually related to performance in Basic Schools – that is, the disregard, in the curricular structures of these courses, of mathematical content knowledge for teaching. Within this scenario, it is necessary to prioritize the beginning of schooling, since it is the fundamental basis of knowledge and impacts the following years of school education. Teaching mathematics is a demanding job, mathematically speaking, and the use that teachers do in Mathematics requires them to know about its specificities in a different way from how other professionals who deal with the subject do (Ball *et al.,* 2005). The need to develop students' conceptual understanding of Mathematics (Kilpatrick; Swafford; Findell, 2001; NCTM, 2012; Brasil, 2018) demands that teachers have in-depth knowledge of Mathematics (Ma, 1999; Ball; Thames; Phelps, 2008).

The literature also points out that many teachers who teach mathematics in the initial years have a history of frustration in Mathematics, lack of self-confidence, fear of failure and

have a higher level of Mathematics anxiety than teachers who work in the final years in Elementary School and High School, because they feel anxious about teaching the most difficult Mathematics content.

This scenario may be related to the lack of preparation felt by the pedagogue precisely because he has generalist training and no training base in teaching specific methodologies. In this sense, understanding and developing the content of the teacher's knowledge is a central way to improve the quality of student learning – through the development of the teacher's knowledge –, which is only possible through training with this specific objective.

Considering that transferring the results of basic research to educational practice is not a new challenge, but a persistent one, in this work we are interested in encouraging discussion about the relationship between development and research projects, with educational practice and with school reality.

In this article, we present and briefly describe two projects that are being carried out in the states of Rio Grande do Norte and Mato Grosso do Sul to expand, deepen and improve knowledge in the teaching of Mathematics, as well as the results observed so far. The first project entitled: *Cognitive Neuroscience and Mathematics Education: a necessary dialogue in teacher training* began in 2021 and is being developed in the Sertão do Seridó region, in the state of Rio Grande do Norte. The other project is entitled *Creation of Children's Literature stories for teaching Mathematics* and began in 2018 and was developed in the Greater Dourados region, in the state of Mato Grosso do Sul. The methodology used involved the analysis of the projects developed.

Our intention in presenting the projects is to reflect on possibilities for how teachers learn mathematics and how they can develop mathematical reasoning in students. Thus, the question that guided this article was: What training actions can contribute more effectively to the development of mathematical and/or didactic knowledge? As partial results, we observed participation in scientific events, recognition of their importance by the school community, preparation of TCC, children's literature books, books with training sequences and we concluded that there is a need and urgency to promote the articulation between teaching and research in training and in work of the Basic Education teacher, bringing education research closer to the two realities that concern it: that of the university and that of the school.

Mathematics Teachers ' Specialized Knowledge -MTSK

Based on the understanding that to achieve a Mathematics teacher profile that breaks with already crystallized ways, it is necessary that teacher training in this area is based on the articulation between theory and practice, between specific knowledge linked to pedagogical knowledge (Ponte, 1992; D' ambrosio, 1996; Cyrino, 2006).

This study, as well as the research projects under analysis, are based on the principle that improvement in Mathematics teaching necessarily, although not exclusively, involves improving teacher preparation and overcoming the problems of initial and continuing teacher training, requiring an analysis of the paradigms that guide such courses (Moriel Junior; Wielewski, 2016).

Therefore, they also consider the studies by Shulman (1986), Ball, Thames and Phelps (2008) and Carrillo *et al.* (2013), among others, who confirm that teachers' knowledge must be different in depth and breadth in relation to the knowledge of other professionals who deal with Mathematics. It needs to be a type of knowledge anchored in specific Mathematics for teaching, which is different from that expected from other professionals.

Assuming that the teacher – and his knowledge – is a factor that has a great impact on the results and learning of students (Nye; Hedges; Konstantopoulos, 2004), it is essential to present here some considerations regarding the knowledge of the teacher who teaches (or teach) mathematics.

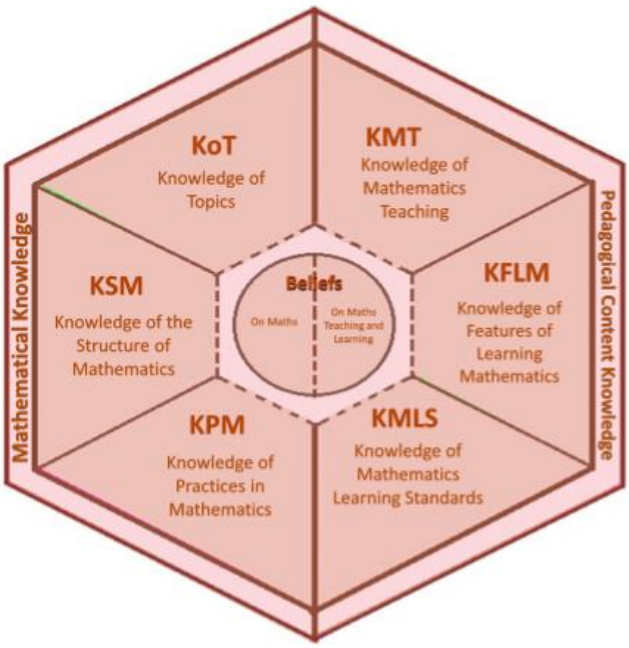
Among the different models resulting from investigations into the knowledge of Mathematics teachers – such as that of Shulman (1986) and that of Ball, Thames and Phelps (2008) –, Mathematics has been developed in recent years Teacher's Specialized Knowledge – MTSK (Carrillo *et al.,* 2018), which is the theoretical framework that underpins the two projects described here.

In this model, the knowledge to teach is considered specialized, with this specialization covering both content aspects and didactic-pedagogical aspects (Carrillo *et al.,* 2013; Carrillo *et al.,* 2018).

The Specialized Knowledge of Mathematics Teachers – MTSK was the model chosen to analyze the data obtained in our study, as well as to analytically investigate the knowledge of (future) teachers who will teach Mathematics, which we will describe below (Carrillo; Climent; Contreras; Muñoz Catalán, 2013; Flowers; Escudero; Carrillo, 2013; Montes *et al.,* 2013; Hills; Contreras; Carrillo, 2013, Carrillo *et al.,* 2018).

This model has two major domains – Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK) – and each of them is subdivided into three subdomains. (See Figure 1)

Figure 1 – MTSK Model



Source: Carrillo- Yañez *et al.* (2018)

We started by talking about the subdomains of Mathematical Knowledge. Topic Knowledge (KoT) is the mathematical content to be taught and its different aspects, that is, KoT is used to describe what and how the teacher knows about the topics he or she will teach. Knowledge of Mathematical Structure (KSM) contains the connections that the teacher makes between mathematical topics, that is, the connections between the contents of different mathematical areas. Specifically, in this subdomain it is possible to identify "temporal connections, which are simplification or complexization, are the relationships that allow us to see elementary content from an advanced point of view and advanced content from an elementary point of view" (Ribeiro; Mamoré; Alencar, 2019, p. 55, our translation). Knowledge of Mathematical Practice (KPM) includes the way of proceeding mathematically, that is, it is about how mathematical knowledge arises.

We move on to the description of the subdomains of Pedagogical Content Knowledge. Knowledge of Mathematics Teaching (KMT) concerns knowledge of the materials and resources available, the way of presenting the content and its characteristics, and may have your own personal theories as a starting point. Knowledge of Learning Characteristics (KFLM)

includes how students learn mathematical content, and encompasses knowledge about errors, obstacles and learning difficulties. It also includes studies of psychological theories to understand student learning.

Knowledge of Learning Standards (KMLS) concerns the knowledge that the teacher has about what students can and should achieve at a given school level, taking into account the curriculum specifications of external bodies. It is worth noting that, to teach Mathematics, graduates in Pedagogy and teachers working in Basic Education cannot rely exclusively on methodological knowledge of teaching Mathematics.

Before proceeding with this work, we believe it is necessary to explain our vision regarding what "evidence" is, according to research and theorists we reviewed. Moriel Junior and Carrillo (2014) define evidence of knowledge as "the verbal, written or attitudinal elements of the subject's manifestation that suggest to the researcher the possibility of certain knowledge having been mobilized, but without providing sufficient and explicit information to guarantee its occurrence".

Methodology - contextualization of studies

After outlining the theoretical framework about the knowledge of the teacher who teaches mathematics, in this section we present that the methodology used to analyze the projects is qualitative and descriptive.

This type of investigation indicates that "descriptive research has as its main objective the description of the characteristics of a given population or phenomenon, establishing correlations between variables." (Gil, 2002, p. 42, our translation). This can be carried out through documents, historical records and others, field studies, surveys, and these must always be supported by variables. In this case, the variables of this analysis will be the different states that present different cultural, regional, structural and human aspects.

To this end, we make a prior explanation of what will be our investigative locus, the research projects: *Cognitive Neuroscience and Mathematics Education: a necessary dialogue in teacher training* and *Creation of Children's Literature stories for Mathematics teaching*, making reference to the context of investigation and to the design and implementation processes of project actions. Next, we describe more specifically the research instruments, as well as the methodological procedures used in data collection and analysis.

PROJECT 1: Cognitive Neuroscience and Mathematics Education: a necessary dialogue in teacher training

This project is based on the most recent results of national and international research that show the need for a change of focus in teacher training and the need for a centrality of discussions around teacher knowledge in an intertwined way with mathematical practices and justifies it if in research on the teaching and learning processes of Mathematics that present in their results the failure of students in learning (Cyrino; Oliveira, 2011; Kaput, 2008; Matos; Ponte, 2009; Stephens; Ribeiro, 2012), at the same time that document the difficulties encountered by teachers in their teaching (Doerr, 2004; Lautenschlager; Ribeiro, 2014; Ponte; Branco, 2013; Ribeiro, 2012; Ribeiro; Cury, 2015; Ribeiro; Oliveira, 2015; Wasserman, 2015).

Statistics have shown the inefficiency of the Brazilian educational system when teaching Mathematics to students. Countries with fewer resources, with lower per capita income and where teachers have worse salaries, are performing better in learning compared to Brazil. When comparing performance by regions, according to the PISA 2018 report, the North and Northeast regions are worse than the Brazilian average. We highlight that there are still few studies that relate the knowledge of neuroscience to mathematical education and therefore one of its main objectives is to promote and expand the development of research that allows for the promotion of collaborative work, creating bridges between the sciences that study the mind and the brain and mathematics education. The objectives of this project are also: (i) to contribute to the continued training of public education professionals and the training of undergraduate students involved in the actions of this project; (ii) use self-regulation and self-efficacy techniques with the aim of increasing the behavioral repertoire of teachers with mathematics learning difficulties and/or mathematics anxiety; and (iii) promote spaces for dialogue and construction of knowledge about the difficulties of Mathematics Learning (DAM), aiming to deepen the concepts of Cognitive Neuroscience and Mathematics Education.

According to Loucks-Horsley (1997), learning opportunities based on teachers' practice need to be designed and developed, with teachers in initial and continuing training, in order to provide professional learning throughout their careers. Therefore, we chose to work with already trained and licensed teachers. We had the participation of 38 participants, of which 17 had already graduated. 10 meetings were held that included moments of theoretical studies on Mind, Brain and Mathematics Education; Training of Teachers who teach mathematics and working moments to carry out tasks involving specialized mathematical knowledge for teaching. Given the participatory nature of the project, we adopted a Design-Based approach

Research (DBR), combining empirical educational research with learning design. In this project, the school is considered a place of learning for teachers and students, but it is also a place of production of knowledge, of practical knowledge. The project was organized in three phases: the first phase involves the recruitment of teachers from the municipal network to join the project and the period of inclusion of higher education participants in teaching units, as well as the development of a shared language and conceptualization of main questions to be addressed. The second phase is the implementation of the intervention project emanating from phase 1 and the third phase consists of improving the project and the proposed artifacts, redirecting the focus of the school community as necessary. At this stage, it is also planned to disseminate the results, expanding to other schools, in addition to holding a Seminar (lecture/discussion).

PROJECT 2 : Creation of Children's Literature stories for teaching Mathematics

This project was created when it was noticed that some teachers had difficulties in teaching mathematics using different methodologies. These difficulties were also noticed in the research by Campos (2007), Garcia Silva (2007) and Alencar (2012) who consider that the teachers' difficulties reflect on the students' learning in Mathematics. Considering that Children's Literature can be one of the ways to make teaching more understandable and enjoyable, we developed this project. To do this, we carried out a national literature review of the main investigations on the topic and found research such as: Cerquetti (2001), Smole (2000), Zacarias and Moro (2005), Reame (2012) Reame (2012) and Gasperin (2013) that they consider working with different methodologies necessary for learning to take place. Given this fact, research that provides teachers with reflection and creates materials can be beneficial for learning actions.

The main objective of the investigation is to identify how the creation of Children's Literature stories (animated e-books and conventional books) for the development of mathematical concepts influence practices and/or professional knowledge of a group of Early Childhood Education and early years teachers. Elementary School.

To carry it out, we used the Design Experiments methodology, based on Cobb, Confrey, di Sessa, Lehrer and Schauble (2003). Therefore, to obtain data for the investigation, we will hold a weekly 4-hour meeting with a group of public teachers from Early Childhood Education and the Early Years of Elementary School, during the duration of the project. The meetings

were held on the premises of the University where the research was carried out and also online, especially during the pandemic years.

Therefore, we carried out 6 steps: i) questionnaire; ii) studies on literature and Mathematics and didactic sequence; iii) creation of children's stories; iv) discussion and analysis of collective constructions; v) creation of illustrations and their analysis; and vi) layout for animated e-books and conventional books.

In this article, we will explain the general results of stages ii and iii as we believe that this could help us answer our questions about: How does a teacher learn mathematics? Or at least Mathematics to be taught to students so that they can enjoy and understand it effectively.

How does the teacher learn? Some possibilities and reflections

When carrying out studies on issues involving Mind, Brain and Mathematics Education, in project 1, we can observe some changes in the attitude of basic education teachers. Knowing, for example, that intense negative emotions can interfere with attention to cognitive processing can lead us to reflect on the image that Mathematics has in Western society and how this image can interfere with learning, since Mathematics is usually seen as scary, difficult, complicated and uninteresting for many people.

Another discussion concerns neuroimaging, which indicates that mathematical reasoning is based on visual processing. The neurobiological basis of mathematical cognition involves complex and dynamic communication between the brain systems of memory, detection and control, and visual processing regions. How can this information contribute to improving mathematics classes? By thinking about it, the (future) teacher is expanding his/her Knowledge of Learning Characteristics (KFLM).

When preparing a class, with the content approach through visual mathematics, the (future) teacher will have to employ the use of visual components in classes such as gold material, folding, games and mathematical software (KMT) in addition to needing to show students students that there are different ways of solving the same problem/exercise (KPM) and that doing it is more important than being the first to finish. In this context, we observe the importance of providing a space providing (future) participating teachers with different moments of individual and collective work and reflection to deepen and expand specialized knowledge for teaching.

In project 2, when we presented studies on the use of children's literature to teach mathematics to teachers, and with this first explanation, it was possible to broaden the horizons of knowledge about new methodologies for teaching and reflecting on Mathematics.

Furthermore, we present six training sequences during the project in these years of development: three carried out in the first year of training (June/2018 to May/2019) and the others in subsequent years. The training sequences used children's books: *The Wolf that Turned into Geometric Shapes* by Edvonete Souza de Alencar and Anttonio Pereira; *Once upon a time there was an armadillo ball*, by Bia Villela; *Love Trigonometry* by Millor; *Little Yellow Riding Hood* by Chico Buarque; *The Massamê Cat and what you see*, by Ana Maria Machado; *Trudi and Kiki,* by Eva Furnari, *A surprise for nine,* by Silvia Regina da Silva Cassimiro and Edvonete Souza de Alencar; *Authored chaos,* by Lilli L'Arronge and Hedi Gnädinger and *The Witch's Soup,* written by Hae Wang Jeong. Some of these books were used together for more than one training sequence.

Therefore, the training carried out could enhance all specialized knowledge to teach mathematics according to Carrillo (2018). Some more evidently, as is the case of KOT and KMT. And this was one of the most discussed, as it allowed the teacher to reflect on new possibilities for teaching Mathematics.

Preparation with training was essential for the team to create children's literature stories. We think that this stage of the investigation is the one that can most contribute to the answer to our investigation in this article, as teachers need, when writing stories, to reflect on the following questions: How to create a story? What characters will we create? What specific content will we cover in the stories? Thus, these questions lead us to consider that they reflect on the KOT, the KSM and the KPM, as teachers needed to think about which content to deal with and how to approach it with an interest in its simplification characteristics, as well as its social use.

It is noted that these questions were essential both for the analysis of other existing stories and for the creation of new stories. To do this, the teacher in training needs to put themselves in the other person's (student's) shoes, reflect on how the child thinks and imagines, what the child's context is like. All these last reflections lead us to believe that teachers are developing KFLM knowledge, as creating material for students requires us to know how they reflect and learn mathematics.

In general, knowledge of KMT and KMLS appears in all construction of the materials, as the proposed curriculum and the benefits and possible teaching situations are consulted at all times.

We therefore consider that promoting training in which the teacher has an active role in their training brings a different model to what we have seen in recent years, in which this was only the receiver of knowledge. In a training process like the one presented, the student is the builder of their knowledge and the trainers are the mediators of reflections.

Some considerations

In view of the above, we consider the teacher as a key element in the teaching-learning process, a reflective professional who makes rational decisions. Therefore, we point out the need and urgency to promote training that allows diverse formats to help teachers learn mathematics and teach the content covered more effectively. We theorize that when the teacher puts himself in the student's place, he or she is able to reflect and think about the main difficulties encountered. Furthermore, it is important that they play an active role in their training.

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