DESIGN OF PHENOMENOLOGICAL MATHEMATICAL TASKS WITH INTEGRATION OF TECHNOLOGY IN THE INITIAL TRAINING OF MATHEMATICS TEACHERS

DESENHO DE TAREFAS MATEMÁTICAS FENOMENOLÓGICAS COM INTEGRAÇÃO DE TECNOLOGIA NA FORMAÇÃO INICIAL DE PROFESSORES DE MATEMÁTICA

DISEÑO DE TAREAS MATEMÁTICAS FENOMENOLÓGICAS CON INTEGRACIÓN DE LA TECNOLOGÍA EN LA FORMACIÓN INICIAL DE PROFESORES DE MATEMÁTICA

Luis Fabián GUTIÉRREZ-FALLAS[1]
e-mail: luisfabian.gutierrez@ucr.ac.cr

How to reference this article:

GUTIÉRREZ-FALLAS, L. F. Design of phenomenological mathematical tasks with integration of technology in the initial training of mathematics teachers. **Revista Ibero-Americana de Estudos em Educação**, Araraquara, v. 19, n. esp. 2, e024074, 2024. e-ISSN: 1982-5587. DOI: https://doi.org/10.21723/riaee.v19iesp.2.18999

| **Submitted**: 02/02/2024
| **Revisions required**: 18/03/2024
| **Approved**: 13/04/2024
| **Published**: 20/07/2024

ARTICLE SUBMITTED TO THE SIMILARITY SYSTEM turnitin

Editor: Prof. Dr. José Luís Bizelli
Deputy Executive Editor: Prof. Dr. José Anderson Santos Cruz

[1] University of Costa Rica (UCR), San José – Costa Rica. Professor and Researcher at the Department of Mathematics Education at the UCR School of Mathematics.

ABSTRACT: Didactic phenomenology is present in school curricula that promote a functional approach to Mathematics, where the resolution of tasks contextualized in real phenomena is sought that make evident the application of Mathematics to understand, interpret and solve problems. The design of these tasks is an activity of the Mathematics teacher, so it is relevant that initial training programs for Mathematics teachers offer opportunities to design tasks that respond to these demands. This text aims to analyze the operationalization of the principles that guide the design of phenomenological mathematical tasks with integration of technology, based on the model of the School Phenomenological Cycle (SPC) in the context of initial training of Mathematics teachers. The results presented here are part of a study that was developed for three years following a qualitative methodology. In this case, the analysis of two mathematical tasks is presented, the results of which show the different mathematical and didactic-mathematical considerations in the design of tasks for the linear function teaching.

KEYWORDS: Initial teacher training. Mathematical tasks. Technology integration. Didactic phenomenology.

RESUMO: *A fenomenologia didática está presente nos currículos escolares que promovem uma abordagem funcional da Matemática, onde se procura a resolução de tarefas contextualizadas em fenómenos reais que tornem evidente a aplicação da Matemática para compreender, interpretar e resolver problemas. O design destas tarefas é uma atividade do professor de Matemática, pelo que é relevante que os programas de formação inicial de professores de Matemática ofereçam oportunidades para elaborar tarefas que respondam a estas exigências. Este texto tem como objetivo analisar a operacionalização dos princípios que norteiam o desenho de tarefas matemáticas fenomenológicas com integração de tecnologia, com base no modelo do Ciclo Fenomenológico Escolar (CFE) no contexto da formação inicial de professores de Matemática. Os resultados aqui apresentados fazem parte de um estudo desenvolvido durante três anos seguindo uma metodologia qualitativa. Neste caso, é apresentada a análise de duas tarefas matemáticas, cujos resultados mostram as diferentes considerações matemáticas e didático-matemáticas no design de tarefas para o ensino de função linear*

PALAVRAS-CHAVE: *Formação inicial de professores de Matemática. Tarefas matemáticas. Integração da tecnologia. Fenomenologia didática.*

RESUMEN: *La fenomenología didáctica está presente en los currículos escolares que promueven un enfoque funcional de las Matemáticas, en donde se busca la resolución de tareas contextualizadas en fenómenos reales que hagan evidente la aplicación de la Matemática para comprender, interpretar y resolver problemas. El diseño de estas tareas es una actividad del profesor de Matemática, por lo que resulta relevante que los programas de formación inicial de profesores de Matemática ofrezcan oportunidades para diseñar tareas que respondan a estas demandas. Este texto tiene el objetivo de analizar la operacionalización de los principios que orientan el diseño de tareas matemáticas fenomenológicas con integración de la tecnología, basado en el modelo del Ciclo Fenomenológico Escolar (CFE) en el contexto de formación inicial de profesores de Matemática. Los resultados aquí presentados forman parte de un estudio que se desarrolló por tres años siguiendo una metodología cualitativa, en este caso, se presenta el análisis de dos tareas matemáticas cuyos resultados evidencian las distintas consideraciones matemáticas y didáctico-matemáticas en el diseño de tareas para la enseñanza de la función lineal.*

PALABRAS CLAVE: *Formación inicial de profesores de Matemática. Tareas matemáticas. Integración de la tecnología. Fenomenología didáctica.*

Introduction

A permanent problem in Mathematics Education is how to design initial teacher training programs that influence the nature and quality of teaching practice, since, according to Hiebert *et al.* (2003), teaching is a cultural practice. According to these authors, teachers learn to teach, in part, by growing up in a culture in which they are passive learners for 12 years or more while they are students at school and, consequently, "faced with the real challenges of the classroom, they often abandon new practices and return to the teaching methods used by their teachers" (Hiebert *et al.*, 2003, p. 201, our translation).

Being conceived as a cultural practice also implies responding to the demands of that culture, so initial teacher training programs must ensure that they meet the demands of the 21st century in terms of developing professional knowledge that is complex and integrative, but also flexible and dynamic, which characterizes a competent and efficient mathematics teacher. For Serrazina (2012), "what training should a teacher have to face all the challenges presented to them? is a question that has received the attention of many trainers and researchers, and to which there is no single answer" (p. 267, our translation).

In particular, Mathematics Education has been significantly transformed through the use of digital technologies with advanced computational, graphical, and symbolic capabilities (Niess, 2012). In this context, international curriculum documents argue that technology plays a critical role in teaching, creating learning environments that "integrate the use of mathematical and technological tools as essential resources to help students learn and make sense of mathematical ideas, reason mathematically and communicate your mathematical thinking" (NCTM, 2014, p. 78, our translation). Thus, the existence, versatility and power of technology lead to a restructuring of what and how students should learn Mathematics, taking into account their preferences and new ways of learning.

Student learning is conditioned by the mathematical tasks proposed by the teacher (Penalva; Llinares, 2011), thus, within this context, mathematical tasks, on the one hand, must demonstrate the applicability of Mathematics for solving tasks contextualized in phenomena that surround the student and, on the other, they must promote the use of efficient technological tools.

This text is part of a study that was developed over three years following a qualitative methodology, in the context of initial training for Mathematics teachers. The approach to the problem lies in the design of tasks to promote the teaching of the Functions theme in high school, tasks that include the application of mathematical knowledge in phenomenological contexts and the integration of technology to explore the task. According to SIMON (2008), the training of Mathematics teachers is a space characterized by its complex nature, however, this also implies a significant relevance in itself in terms of understanding the elements present in the training of future Mathematics teachers.

> Mathematics teacher training is more difficult and complex than Mathematics Education, as it encompasses the latter. Likewise, research into Mathematics teacher training is more difficult and complex than research in Mathematics Education (Simon, 2008, p. 27, our translation).

The objective of this text is to analyze the operationalization of the principles that guide the conception of phenomenological mathematical tasks with the integration of technologies, based on the school phenomenological cycle model in the context of initial training of Mathematics teachers. In this case, the analysis of two mathematical tasks is presented, the results of which show the different mathematical and didactic-mathematical considerations in the design of tasks for teaching linear function.

Initial training for Mathematics teachers

In the last two decades, international research and guidelines have demanded greater emphasis on Mathematics Education, promoting, among other things, mathematical reasoning, problem solving and the use of technology (NCTM, 2014). This context requires changes for many teachers, related to their beliefs, attitudes and professional knowledge (Swars *et al.*, 2009). Therefore:

> These changes must begin during teacher training programs in the contexts of mathematical content courses, teaching methodologies, and field experiences in schools. Consequently, changes in beliefs, attitudes and knowledge in these contexts must be identified, fully understood, appropriately emphasized and routinely measured throughout courses as important outcomes of teacher education programs (Swars *et. al*., 2009, p. 48, our translation).

It is from the initial training of Mathematics teachers that the path must be traced for these changes to happen. Recognized as a process of learning to teach (Llinares, 1998), initial teacher training is a process influenced by several factors, among which four main ones stand out: (i) *professional knowledge* that future teachers develop in their training program; (ii) the *processes of reflection* on your future professional practice and the different tasks you perform; (iii) the *conceptions and beliefs* that future teachers have about teaching and learning

Mathematics and (iv) the *context* of their future professional practice (Llinares, 1998; Obter; Chapman, 2008; Swars *et al.*, 2009).

In Mathematics Education, it is well discussed and argued that the teacher's *professional knowledge* is practical, complex, integrative, critical and professionalized knowledge (Gómez; Rico, 2004; Ponte, 2012; Obter; Chapman, 2008). As for its nature, the professional knowledge of the Mathematics teacher is *theoretical-practical knowledge*, which develops from theoretical exploration and mobilization for experience in practice, that is, the teacher has the need to analyze theoretical knowledge in the face of singularity of the situations that occur in their professional practice, highlighting that this analysis implies a *process of reflection* on the part of the teacher about their action, thus allowing them to build knowledge about the practice. In this regard, Ponte (2012) considers that

> The professional knowledge of the Mathematics teacher includes several aspects, of which we are especially interested in teaching practice, the one in which the specificity of the Mathematics discipline is most strongly felt, and which we call didactic knowledge (Ponte, 2012, p. 86-87, our translation).

For Gómez and Rico (2004), didactic knowledge "is the knowledge necessary to organize teaching and learning activities" (p. 4). These authors identify three domains that integrate didactic knowledge: (i) knowledge about the curriculum as a global planning and structuring tool; (ii) knowledge about the foundations of school mathematics (mathematics, learning, teaching and assessment); and (iii) knowledge about Mathematics Education, that is, knowledge about conceptual and methodological tools for lesson planning. Likewise, Ponte (2012) defines didactic knowledge of Mathematics based on four main domains: (i) knowledge of Mathematics, (ii) knowledge of the curriculum, (iii) knowledge of the student and their learning processes, and (iv) knowledge of work processes in the classroom (knowledge of teaching practice). The position of these authors coincides with the conceptualization made by Azcárate (2004) when characterizing the teacher's professional knowledge as complex and integrative.

Regarding the *context* of future professional teaching practice, on the one hand, it must be considered that "in teacher training it is not enough to think about what should be taught, it is also necessary to think about how to teach" (Serrazina, 2012, p. 267-268, our translation). In this way, initial training must "provide future teachers with opportunities that allow them to understand, appreciate and embrace the complexity of their practice as a basis for ongoing training" (Ponte; Chapman, 2008, p. 256, our translation).

On the other hand, this context presents new demands related to the advancement of technology in the 21st century and, according to curricular guidelines, Mathematics Education must prioritize the promotion of teaching and learning Mathematics with technology (AMTE, 2017; NCTM, 2014). Thus, new questions were added in relation to the professional knowledge of the Mathematics teacher, for example: How does the teacher develop his technological knowledge? How does the teacher articulate technological knowledge with mathematical knowledge and didactic-mathematical knowledge? (Koehler *et al.*, 2014).

In this scenario, when technology is integrated, new domains of professional knowledge emerge as a result of the articulation of didactic-mathematical knowledge with technological knowledge, giving rise to models such as TPACK [2](Gutierrez-Fallas; Henriques, 2021; Koehler *et al.*, 2014). The TPACK model is a dynamic and flexible framework suitable for defining and characterizing the type of knowledge that a teacher needs to develop to effectively integrate technology into the teaching and learning of Mathematics. According to Mishra and Koehler (2006), TPACK is defined as an integrative knowledge that results from the simultaneous articulation of content, pedagogy and technology, which the teacher must demonstrate.

To this end, it is necessary that training programs guide future teachers in learning new technologies and their efficient use in proposals for teaching mathematical content. This formative learning is a process of acquiring technological knowledge and its articulation with didactic knowledge, considering how these technologies can affect teaching strategies, the school curriculum itself and the way students explore and learn mathematical content (Niess, 2012)

Mathematical tasks with technology integration

Mathematical tasks are action proposals that teachers propose to their students to promote the learning of mathematical content, that is, a mathematical task conditions what students will do with that task and delimits what they can learn (Penalva; Llinares, 2011). The purpose of a school mathematical task is its resolution, in this sense the activity is defined as "the set formed by the task and the system of individual and/or social cognitive activities developed by the solver" (Penalva; Llinares, 2011, p. 28, our translation).

The mathematical task makes it possible to establish a link between teaching and learning. This bond is guided by teaching when the teacher defines the aims and objectives to

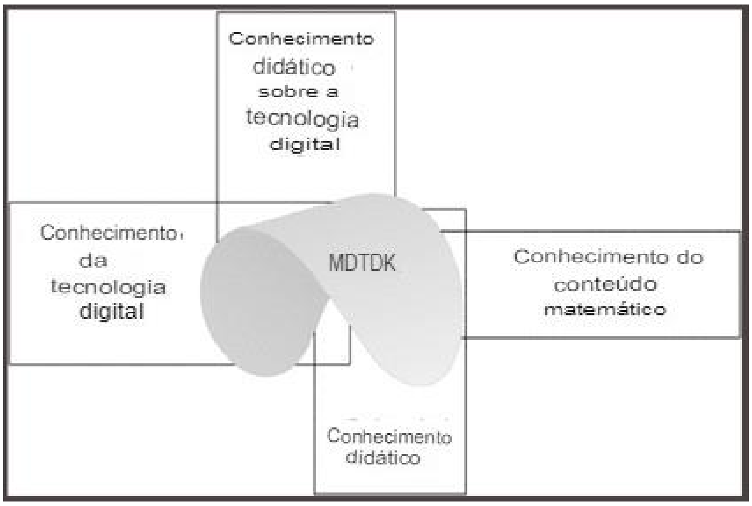
[2] *Technological Pedagogical Content Knowledge*

be achieved, taking into account the curriculum to which he responds, the context and student population he addresses, the type of task and the cognitive level of the task. (Ponte, 2005). In terms of learning, the mathematical task should "allow students to think about mathematical situations, rather than remembering recipes that they must follow" (Penalva; Llinares, 2011, p. 30, our translation).

According to Ponte (2005), tasks are the basis of the mathematical activity that a student can develop during their solution; the author defines four types of tasks according to their degree of challenge and their degree of structure: exercises, problems, investigations and explorations. Reduced challenge tasks (low cognitive demand) are exercises and explorations, while *exercise* is a routine task, which is solved with an immediate procedural strategy and a single correct answer is achieved, *exploration* is an open task, where there is a certain indeterminacy of what is given or what is requested in the statement and the resolution of a scan admits several valid answers. For Ponte (2005), solving a *problem* implies the adaptation of some strategy or the application of a set of procedures to reach a valid answer, as it promotes the connection of mathematical concepts within a real or fictitious context associated with a phenomenon of everyday or social life; Research is an open *task* that can be constructed together with students, giving them a greater degree of responsibility in formulating strategies for carrying out the research.

However, in a context where technological tools are integrated in the design and resolution of mathematical tasks, it is necessary to consider some elements. Leung (2017) discusses the integration of technology into tasks used in the Mathematics classroom and defines *Technopedagogical Task Design* as the design of tasks for "pedagogical processes in which students are provided with expanded abilities to explore, reconstruct (or reinvent)) and explain mathematical concepts using embedded tools in a technology-rich environment" (Leung, 2017, p. 327, our translation). This author adapts the TPACK model (Mishra; Koehler, 2006) and describes a type of knowledge called *Mathematics Digital Task Design Knowledge* (MDTDK) that arises from the intersection of four knowledge: (i) knowledge of mathematical content; (ii) knowledge of the technological tool; (iii) didactic knowledge of Mathematics; and (iv) didactic knowledge of the technological tool (Figure 1).

Figure 1 – *Knowledge for the Digital Mathematical Tasks Project*



Source: (Leung, 2017, p. 7)

The author further adds that "MDTDK is flexible in the sense that it should not be a rigid knowledge structure and is susceptible to change as interactions between the four knowledge domains evolve" (Leung, 2017, p. 6, our translation). For example, in the study by Gutiérrez-Fallas and Henriques (2018), this framework was operationalized with future mathematics teachers, the results showed that future teachers mobilize *didactic knowledge of the technological tool* in the design of tasks, conceiving technology from a didactic perspective, mainly as a motivating, dynamic and innovative resource; Future teachers recognize not only the potential of the technological resource, but also the way in which these potentials associated with a didactic intention contribute to the exploration of mathematical content and the development of student learning in solving the task. The articulation of *knowledge of the technological tool* with *didactic knowledge of Mathematics* was also evident, as future teachers identified the options offered by the technological tool and guided its use for the development of mathematical reasoning, independent and collaborative work between peers.

Didactic phenomenology

Hans Freudenthal (1983) argued that mathematics is a cognitive and public knowledge instrument for organizing, structuring and mathematizing parts of reality. For this author, it is through this organization, structuring and mathematization that each individual personally appropriates mathematics; Therefore, he argues that it is through teaching that these phenomena should be sought in the students' environment that are associated with the mathematics they are learning.

Freudenthal (1983) distinguishes four types of phenomenology: (i) *pure phenomenology*, which are the phenomena that are organized in mathematics taken in their current state and considering their current use; (ii) *didactic phenomenology*, involving the phenomena present in the students' world and those proposed in the teaching and learning sequences; (iii) *genetic phenomenology*, phenomena are considered in relation to the cognitive development of learners, and (iv) *historical phenomenology*, special attention is paid to the phenomena for which the concept in question was created and how it spread to other phenomena.

These ideas were the basis of several studies (Gómez; Cañadas, 2011; Gutiérrez-Fallas, 2023) allowing the contributions of didactic phenomenology to be increasingly considered in school curricula. For example, the school curriculum in Mathematics can be viewed from four approaches (Rico; Lupiañez, 2008): instrumental or technological approach, structural or technical approach, functional approach and integrated approach. In this text I will refer to the functional approach, as the curricular approach that adjusts to the considerations of didactic phenomenology, since the functional approach promotes contextualized teaching in real situations and the resolution of problem-type tasks constitutes a main element in this approach.

Internationally standardized tests, such as PISA (OECD, 2010), encourage the use of this approach to teaching Mathematics, where the student is able to use mathematical knowledge to solve problems in different situations. Specifically, it is argued that the following should be developed in the student:

> an ability to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using concepts, procedures, facts and tools to describe, explain and predict phenomena. It helps individuals recognize the role of mathematics in the world and make the well-informed judgments and decisions necessary for constructive, engaged and reflective citizens (OECD, 2006, p. 4, our translation).

To respond to these guidelines, Gómez and Cañadas (2011) propose a process to be developed in the initial training of Mathematics teachers called Phenomenological Analysis, which consists of the description of the phenomena that organize school mathematics and are related to the corresponding mathematical concept or structure. This analysis "begins by delimiting the situations in which the mathematical concepts involved are used, those in which they show their functionality" (Lupiañez, 2009, p. 48, our translation).

Cañadas and Gómez (2013, p. 34) present a series of questions that Phenomenological Analysis must answer, including: What phenomena give meaning to this mathematical content?

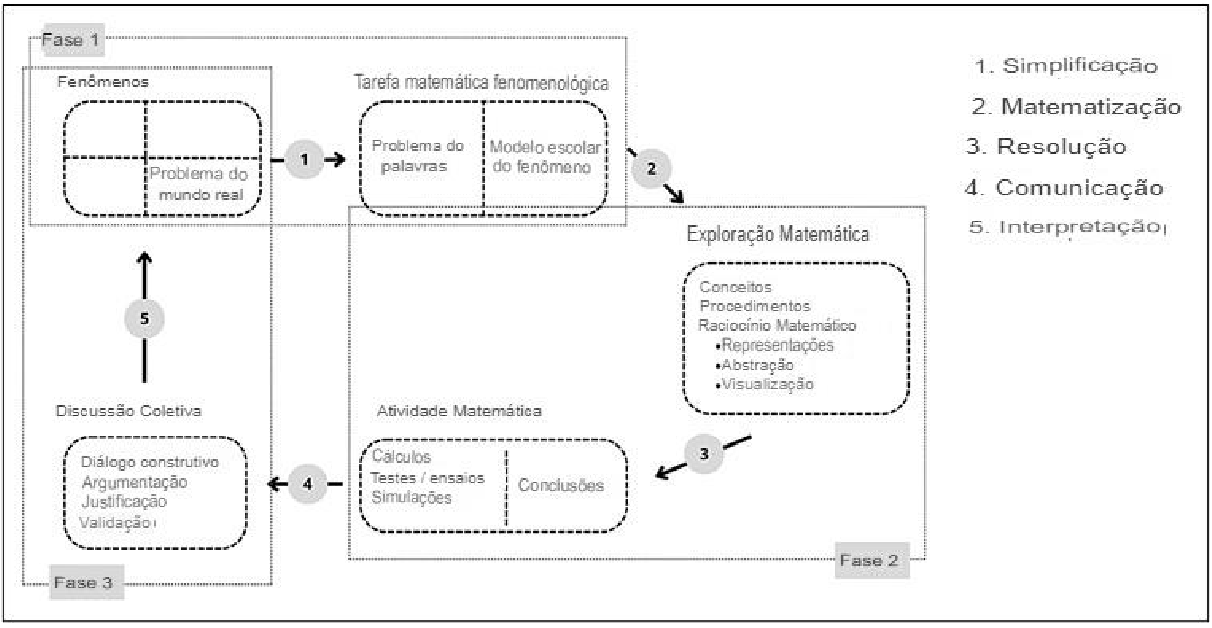
What is this mathematical content for? What problems does it answer? What characteristics do the phenomena that give meaning to this content share?

To carry out a Phenomenological Analysis, Gómez and Cañadas (2011) propose that the future Mathematics teacher: investigate the possible real phenomena associated with the content, group the phenomena in contexts according to their relevant structural characteristics from a mathematical point of view, design tasks simplifying the phenomenon for a word problem that can be presented to students, Manage the execution of the task in the classroom and discuss with students the results obtained when solving the task. This process is called *Phenomenological Analysis Cycle from a School Perspective* (Gómez; Cañadas, 2011, p. 80).

Based on this model, for three years the stages of this cycle have been mobilized in the training of future Mathematics teachers in terms of designing tasks for teaching and learning functions (Gutiérrez-Fallas, 2023). With this experience, an adaptation to the cycle was made (Figure 2) and, in addition, five principles are proposed that guide the project and conceptualize phenomenological mathematical tasks with the integration of technology.

The *School Phenomenological Cycle* (CFE) is made up of three phases. *Phase 1 – Simplification*, consists of two moments in which the teacher is responsible, a first moment is the investigation of phenomena that problematize real-world situations associated with the mathematical content to be taught and a second moment in which the teacher simplifies this phenomenon and draws a phenomenological mathematical task proposing a school model of the phenomenon. *Phase 2 – Mathematization and Resolution*, consists of the mathematical exploration of the task by the student, mobilizing their mathematical reasoning and using the conceptual-procedural knowledge they have to mathematize the situation, this gives rise to the resolution of the task and thus develop their mathematical activity, allowing them to obtain the corresponding conclusions and mathematical results. *Phase 3 – Communication and Interpretation*, consists of a moment of collective discussion, where the teacher leads a constructive dialogue inviting students to argue and justify their results, so that they finally allow the conclusions to be validated in light of the interpretation of the phenomenon that gave rise to the problem posed in the task.

Figure 2 – School phenomenological cycle



Source: Written by the author

Regarding the design principles of phenomenological mathematical tasks with technological integration, I present below five guiding principles for a Mathematics teacher (or future teacher) to develop a task according to the *CFE.*

- **P1.** The task is linked to a real phenomenon.
- **P2.** The school model of the phenomenon is appropriate to the curricular level at which the task is directed.
- **P3.** The technological tool enhances mathematical exploration and activity for visualizing mathematical content in the development of mathematical reasoning.
- **P4.** The technological tool promotes autonomous work and motivates student collaboration.
- **P5.** What is required by the task calls for argumentation and justification of the results, as well as the interpretation of these results in coherence with the real phenomenon.

Methodology

The study from which this article derives is located in the interpretative paradigm with a qualitative approach, carried out in a context of initial training of Mathematics teachers, in which the author of the text was the researcher and the teacher trainer of the program.

The interpretative paradigm values the understanding of meanings seeking to penetrate the personal world of the subjects, within a context in which interaction occurs between the researcher and the researched (double hermeneutics) and where the production of knowledge is inductive, interactive and spiral (Coutinho, 2011). According to the objective of the study and the nature of the context in which the data was collected, it was necessary for the researcher to interpret, clarify and describe the data collected according to the participants involved. As a trainer, it was essential to be part of the environment where this data is produced and collected: an initial training course for Mathematics teachers.

The study was developed over a period of three years (2020, 2021, 2022) and the experience took place in a course in an initial training program for Mathematics teachers at a university in Costa Rica. This course is the third year of the program, it is didactic-mathematical in nature and among its training objectives are to develop didactic knowledge on the topic of Functions for secondary education, in particular, one of the course contents is the didactic phenomenology of the discipline of Functions within the school curriculum in Costa Rica. To develop this content, future teachers were asked to carry out a phenomenological analysis of mathematical content associated with the theme *Functions* and design phenomenological mathematical tasks for teaching and learning this content, integrating technological tools for both the conception and resolution of tasks.

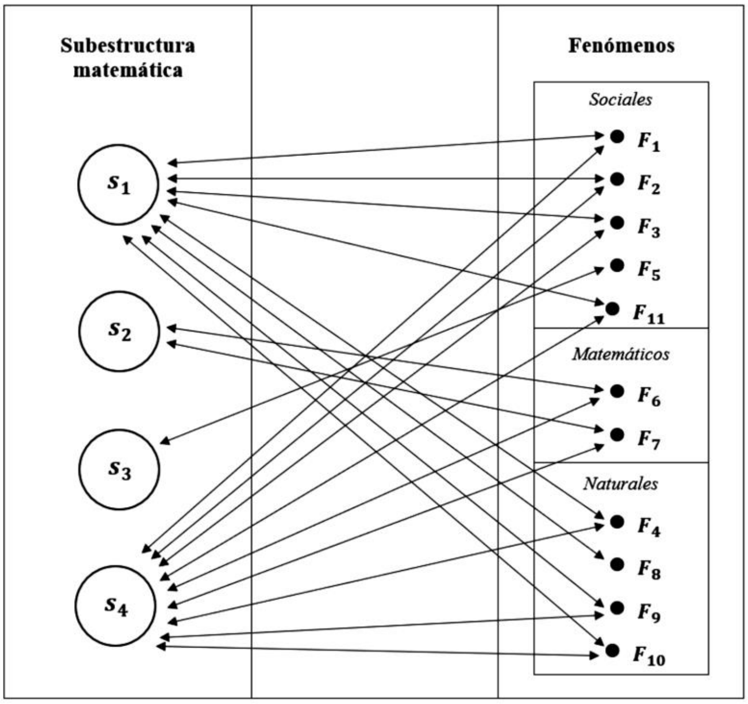
Data collection was carried out based on the productions of the future teachers, in this case these productions consisted of the tasks they prepared and, in addition, evidence of their written reflections on the work carried out was collected. Data analysis was carried out in a descriptive and interpretative way, with the aim of highlighting the operationalization of the five principles of the project in the tasks prepared by future teachers within the CFE model. For the purposes of this article, an analysis of two of the tasks developed by a couple of future teachers who worked collaboratively on the design of these tasks is presented. To guarantee anonymity, fictitious names are given to future teachers, called José and Ana.

The objective of the results presented in this text is to highlight the elements mobilized by José and Ana in the design of phenomenological mathematical tasks for teaching and learning Linear Function, so that there are no elements associated with the execution of these tasks in the school classroom. To achieve this objective, the two tasks prepared by future teachers were analyzed based on the definition of the five principles mentioned in the previous section.

Main results

Before preparing the phenomenological mathematical tasks, José and Ana carried out a phenomenological analysis of the topic of Linear Function. In this analysis, they defined four mathematical substructures of this topic, recognized eleven real phenomena, organized the phenomena into three contexts: social, mathematical and natural, and established the relationship of these phenomena with each of the substructures (Figure 3).

Figure 3 – Scheme of the Phenomenological Analysis of the Linear Function



Source: Prepared by José and Ana

The four defined substructures are: S1. A linear function whose graphical representation does not contain the origin; S2. Linear function whose graphical representation contains the origin; S3. Constant linear function; and S4. Linear Equation. The eleven phenomena are listed below:

- F1. The cost/quantity relationship of producing a product.
- F2. The relationship between wages and quantity of products sold.
- F3. The relationship between time and quantity.
- F4. The relationship between an animal's heart rate and its body temperature.
- F5. The relationship between a fixed salary and the quantity of products sold.
- F6. The relationship between the perimeter and the side measurement of regular polygons.
- F7. The relationship between the length of the circle and the measurement of its radius.
- F8. The relationship between different units of temperature measurement.

- F9. Relationship between bone measurement and height of men and women.
- F10. The relationship between the length of part of an animal's body and its entirety.
- F11. The relationship between price and supply or demand for a product.

Subsequently, José and Ana designed five phenomenological mathematical tasks, in which they used three main technological tools for their conception: the website *wix.com, the genial.ly* platform and the *GeoGebra* software. The results of the design analysis of two of the tasks developed by José and Ana, T1 and T2, are presented below. These results are organized based on the five design principles proposed earlier in this text.

P1. *The task is linked to a real phenomenon.*

Regarding T1, it corresponds to the linear model that relates a person's age to their heart rate, in this case, the criterion function, where it represents the person's age in years and the maximum heart rate in beats per minute (bpm). While T2 is a task associated with an aeronautical engineering phenomenon, specifically, the linear relationship that can be established between the flight time of a Boeing 727 aircraft and fuel consumption, taking as a reference that a Boeing 727 uses approximately 4,850 liters of fuel for each hour of flight.$f(x) = 220 - xxf(x)$

In both tasks (Figure 4), there is evidence of a significant process of investigating real phenomena that can be modeled by the criterion of a linear function. For José and Ana, this search "provides the teacher with a series of ideas that allow him to create tasks that enhance not only students' interest in this subject, but also the application of knowledge and its emergence in the world around them" (Re [3]).

Thus, there is evidence of an approach to real-world phenomena, the understanding of that phenomenon and the elements necessary to design a phenomenological mathematical task that allows the exploration of the corresponding mathematical content, in this case, to explore the Linear Function.

[3]The Re symbology will be used to refer to **José and Ana's Written Reflection on the design of the tasks.**

Figure 4 – Proposed tasks T1 and T2 on *genial.ly*

Source: Prepared by José and Ana.

Q2. *The school model of the phenomenon is appropriate to the curricular level at which the task is directed.*

Both tasks are aimed at the secondary level of the Costa Rican educational system, with the aim of recognizing the linear relationship between two variables, the different representations that allow this relationship to be modeled and the main characteristics of the Linear Function.

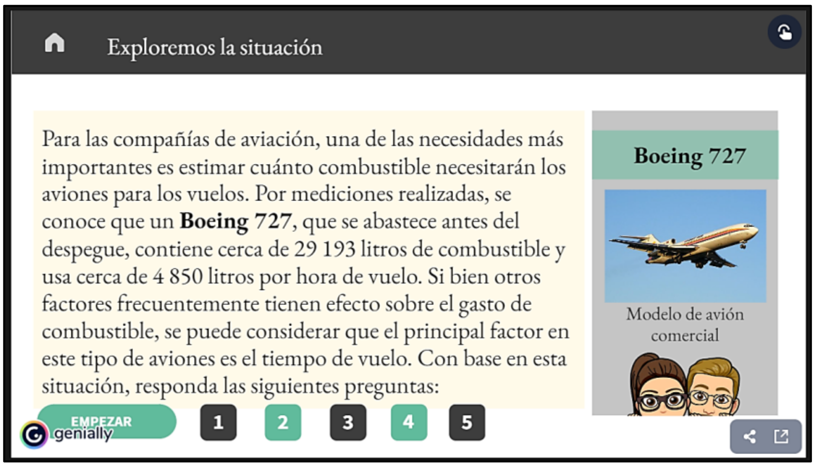
In T1 (Figure 5) José and Ana write a word problem where they combine textual representation with symbolic-algebraic representation to define the school model of the phenomenon, placed in the context of physical activity that a person can perform for the benefit of their health. This statement is very close to the immediate reality of many school teenagers who play a sport or perform some physical performance in their daily lives. Furthermore, it shows proximity to a curriculum that promotes people's physical health.

Figure 5 – Statement T1

Source: Prepared by José and Ana.

Regarding T2 (Figure 6), José and Ana's statement only shows the use of alphanumeric textual representation to establish the school model of the phenomenon in which the task will be developed. In this case, taking into account that, although many students have never traveled by plane, the curiosity that thinking about such a possibility provokes is commonly recognized, so this task appeals to the interest it can generate in students. In this regard, José and Ana reflect that "the design of mathematical tasks allowed us to better understand how to approach this topic through situations or problems with a particular context, in order to correctly teach the topic of linear function" (Re).

Figure 6 – Instruction T2



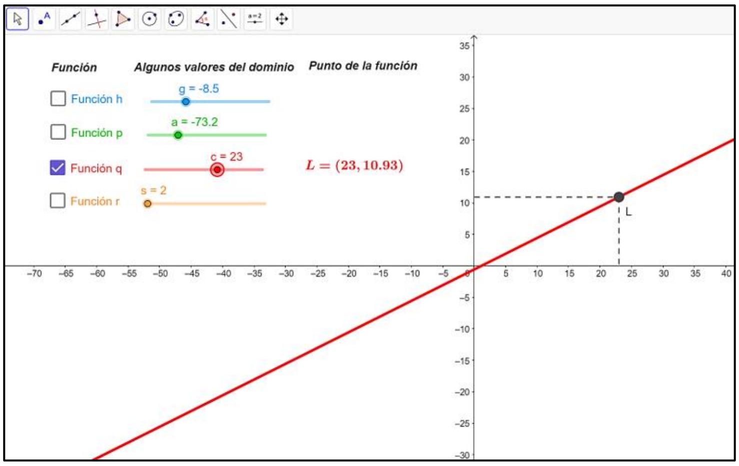
Source: Prepared by José and Ana.

Q3. *The technological tool enhances mathematical exploration and activity for visualizing mathematical content in the development of mathematical reasoning.*

José and Ana used two main tools to promote exploration and mathematical activity, on the one hand, they used *genial.ly* to create the dynamic sequence of the task and the problems associated with it, for example, in Figure 5 and Figure 6 it is possible see the different navigation buttons represented by numbers that guide the exploration of the task throughout its resolution.

On the other hand, they used GeoGebra (Figure 7) with the purpose of consolidating the results obtained during the mathematical activity of the task in terms of the characteristics of the Linear Function, allowing the visualization of the different elements of the mathematical content in a dynamic environment offered by the software.

Figure 7 – Example of using the GeoGebra resource in T1



Source: Prepared by José and Ana.

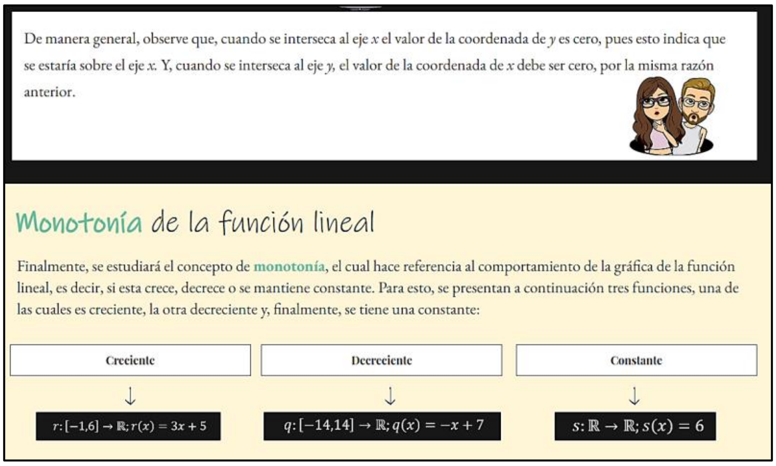
The integration of these technological tools by José and Ana shows the recognition that these resources enhance the teaching of mathematical content and bring significant benefits in terms of student motivation and learning. For future teachers, it is relevant that "there are more and more tools that mathematics teachers have for developing useful lessons that promote student learning" (Re).

Q4. *The technological tool promotes autonomous work and motivates student collaboration.*

According to their reflections, José and Ana state that the process involved in designing tasks also allowed them to recognize how to take them to a class with the intention of improving understanding of mathematical content, arguing that "through tasks such as designing mathematical tasks, it is possible to know how to use them in a class, the large number of concepts that are immersed in them, which allows the development of a better understanding of the mathematical topic treated" (Re).

Part of this consists of promoting students' autonomous work while solving the task, for which future teachers used the *wix.com website* to host a virtual learning environment that guides the student independently (Figure 8). This site presents very detailed instructions, inviting the student to navigate through different options associated with the tasks, their resolution, and the mathematical results that are discovered as the tasks are solved.

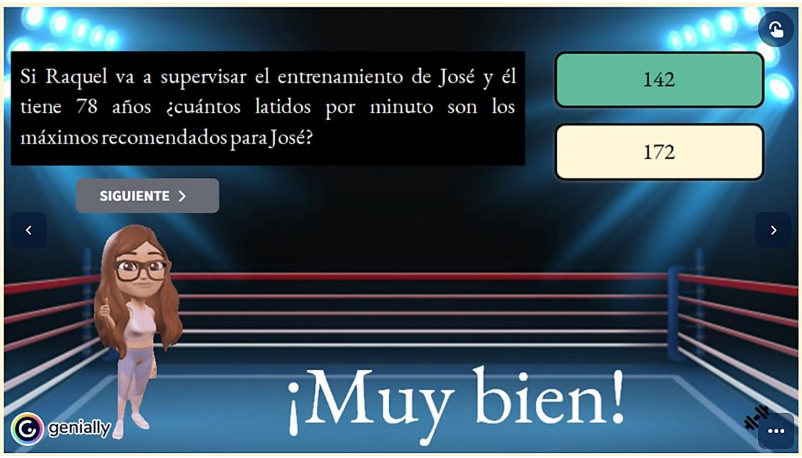
Figure 8 – Example of using *wix.com*



Source: Prepared by José and Ana.

Furthermore, José and Ana integrated elements that maintained motivation to carry out the task and work collaboratively with their peers. For example (Figure 9), they made use of the *genial.ly* platform to introduce iconic animated elements that would allow establishing a link with the technological tool while solving the task.

Figure 9 – Example of interaction with the technological tool in T1



Source: Prepared by José and Ana.

***Q5.** What is required by the task calls for argumentation and justification of the results, as well as the interpretation of these results in coherence with the real phenomenon.*

The questions to be resolved in the task are the engine of the student's exploration and mathematical activity. The richness of these questions becomes evident as argumentation, justification and interpretation of the results obtained are promoted. The tasks designed by José and Ana show a very significant potential for students to establish solid arguments based on their mathematical knowledge, justify their answers by referring to the procedures developed and, at the same time, interpret their answers in light of the phenomenon at hand that the task refers to.

For example, Figure 10 shows a question in which students must not only carry out the respective calculation of the fuel needed to make a 7-hour flight, but also interpret this value within the context defined by the school model of the phenomenon that was presented with the task. Furthermore, José and Ana recognize the contribution that promoting tasks of this nature brings to their training, reflecting that:

> It is possible to affirm that, thanks to the design of didactic proposals for the linear function, it was possible to grow substantially as professionals, generating new knowledge and valuable learning that, without a doubt, will improve Mathematics Education in Costa Rica (Re).

Figure 10 – Example of interaction with the technological tool in T1

Source: Prepared by José and Ana

Conclusions

As evidenced in school curricula, for example, in the Costa Rican curriculum, didactic phenomenology is positioned as a fundamental piece to promote the functional approach to mathematics. Therefore, integrated into the initial training of Mathematics teachers, didactic phenomenology develops professional skills and competencies associated with the analysis, investigation and systematization of information, critical thinking, creativity and educational innovation.

This is enhanced when didactic phenomenology occurs in teacher training programs as an effective and efficient tool for teachers to recognize phenomena that respond to the mathematical content to be taught and design school mathematical tasks within these phenomenological contexts.

The results presented here show that the design of phenomenological mathematical tasks with the integration of technology constitutes an experience that allows future teachers to consolidate their didactic-mathematical knowledge and visualize mathematical concepts in a dynamic way as the contexts of application of these concepts are explored. According to the results, these formative experiences are also enriched by the conceptions that future teachers have about the potential that technology offers for understanding mathematical content when solving tasks.

for the Design of Digital Mathematical Tasks was evident, in particular the results showed that José and Ana not only mobilized their technical knowledge of the digital tools they used to design the tasks, but also didactic knowledge of the tool, which allowed future teachers to create an environment conducive to learning Linear Function.

It is concluded that the CFE constitutes a path for the design and implementation of phenomenological mathematical tasks. However, this path may present some obstacles, for example, three are considered: (i) the time we have in initial teacher training courses *versus* the number of countless topics to be covered, (ii) the conceptions and beliefs of future teachers about the functional approach to Mathematics and (iii) the distance that exists in relation to other scientific areas, social or everyday situations that can make it difficult to understand real phenomena in contexts little known by future mathematics teachers.

However, initial training programs must continue to problematize teaching and learning situations, offering opportunities for future Mathematics teachers to mobilize their professional knowledge in an articulated way, particularly in the design of phenomenological mathematical tasks with the integration of technology, allowing the mobilization of their mathematical, didactic-mathematical and technological knowledge.

REFERENCES

ASSOCIATION OF MATHEMATICS TEACHER EDUCATORS (AMTE). **AMTE technology position statement**: preparing teacher to use technology to enhance the learning of mathematics. Homepage. Raleigh, NC, 2006.

CAÑADAS, M.; GÓMEZ, P. **Apuntes sobre análisis de contenido**. Módulo 2 de MAD. Bogotá: Universidad de los Andes, 2013.

COUTINHO, C. **Metodologia de investigação em ciências sociais e humanas, teoria e prática**. Coimbra: Edições Almedina. 2011.

FREUDENTHAL, H. **Didactical Phenomenology of Mathematical Structures**. Dordrecht: Reidel, 1983.

GÓMEZ, P.; CAÑADAS, M. C. La fenomenología en la formación de profesores de matemáticas. **Voces y Silencios. Revista Latinoamericana de Educación**, *[S. l.]*, v. 2, n. especial, p. 78–89, 2011. DOI: 10.18175/vys2.especial.2011.05.

GÓMEZ, P.; RICO, L. Integration of didactical knowledge and mathematical content knowledge in pre-service teacher training. *In*: **ICME**, 10., 2004. **Anais** […]. Copenhague, Dinamarca, 2004.

GUTIÉRREZ-FALLAS, L. F. Taller: Diseño de tareas matemáticas fenomenológicas para la enseñanza y el aprendizaje de las Funciones. *In*: CIAEM, 16., 2023. **Anais** […]. Lima, Perú, 2023.

GUTIÉRREZ-FALLAS, L. F.; HENRIQUES, A. (2018). O TPACK de futuros professores na adaptação de tarefas matemáticas. *In*: A. PEDRO, J. PIEDADE, J. F. MATOS, N. DOROTEA; N. PEDRO (org.). **Atas do V Congresso Internacional das TIC na Educação**. Lisboa: Instituto de Educação da Universidade de Lisboa, 2018. p. 553-565.

GUTIÉRREZ-FALLAS, L. F.; HENRIQUES, A. Princípios de design de uma experiência baseada no TPACK na formação inicial de professores de matemática. **Zetetike**, Campinas, SP, v. 29, n. 00, e021006, 2021. DOI: 10.20396/zet.v29i00.8661780.

HIEBERT, J., MORRIS, A. K.; GLASS, B. Learning to learn to teach: An "experiment" model for teaching and teacher preparation in mathematics. **Journal of Mathematics Teacher Education**, [*S. l.*], v. 6, 2003, pp. 201-222.

KOEHLER, M.; MISHRA, P.; KERELUIK, K.; SHIN, T. S.; GRAHAM, C. The technological pedagogical content knowledge framework. *In*: SPECTOR, J.; MERRILL, M.; ELEN, J.; BISHOP, M. (org.). **Handbook of research on educational communications and technology**. New York, NY: Springer, 2014. p. 101-111.

LEUNG, A. Exploring techno-pedagogic task design in the mathematics classroom. *In:* LEUNG, A.; BACCAGLINI-FRANK, A. (org.). **Digital technologies in designing mathematics education tasks**: potential and pitfalls. Cham: Springer, 2017. p. 3-16.

LLINARES, S. Aprender a enseñar Matemática en la enseñanza secundaria: Relación dialéctica entre el conocimiento teórico y práctico. **Revista Interuniversitaria de Formación del Profesorado**, [*S. l.*], v. 32, 1998. p. 117-127.

LUPIAÑEZ, J. **Espectativas de aprendizaje y planificación curricular en un programa de formación inicial de profesores de matemáticas de secundaria**. 2009. Tesis (Doctoral) – Universidad de Granada, Granada, 2009.

MISHRA, P.; KOEHLER, M. J. Technological pedagogical content knowledge: a framework for integrating technology in teachers' knowledge. **Teachers College Record**, [*S. l.*], v. 108, n. 6, p. 1017-1054, 2006

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM). **Principles to actions: Ensuring mathematical success for all**. Reston, VA: NCTM, 2014.

NIESS, M. L. Rethinking pre-service mathematics teachers' preparation: technological, pedagogical and content knowledge (TPACK). *In*: POLLY, D.; MIMS, C.; PERSICHITTE, K. (org.). **Developing technology-rich, teacher education programs**: key issues. Hershey, PA: IGI Global, 2012. p. 316-336.

OCDE. **PISA 2006**. Marco de la Evaluación. Conocimientos y habilidades en Ciencias, Matemáticas y Lecturas. Madrid: Ministerio de Educación, 2006.

PENALVA, M. C.; LLINARES, S. Tareas matemáticas en la educación secundaria. *In*: GOÑI, J. M. (ed.). **Didáctica de las matemáticas**. GRAO: Barcelona, 2011. p. 27-74.

PONTE, J. P. Estudando o conhecimento e o desenvolvimento profissional do professor de matemática. *In*: PLANAS, N. (org**.). Educación matemáticas**: teoría, critica y prática. Barcelona: Graó, 2012. p. 83-98.

PONTE, J. P. Gestão curricular em Matemática. *In*: ASSOCIAÇÃO DE PROFESSORES DE MATEMÁTICA (APM). Grupo de Trabalho de Investigação (GTI) (org.). **O professor e o desenvolvimento curricular**. Lisboa: APM, 2005. p. 11-34.

PONTE, J. P.; CHAPMAN, O. Preservice mathematics teachers' knowledge and development. *In*: ENGLISH, L. (org.). **Handbook of international research in mathematics education**. 2. ed. New York, EUA: Routledge, 2008. p. 225-263.

RICO, L.; LUPIÁÑEZ, J. L. **Competencias matemáticas desde una perspectiva curricular**. Madrid: Alianza Editorial, 2008.

SERRAZINA, L. Conhecimento matemático para ensinar: papel da planificação e da reflexão na formação de professores. **Revista Eletrônica de Educação***,* [*S. l.*], v. 6, n. 1, p. 266-283, 2012.

SIMON, M. The challenge of mathematics teacher education in the area of mathematics education reform. *In*: JAWORSKI, B.; WOOD, T. (org.), **International Handbook of Mathematics Teacher Education, The Mathematics Teacher Educator as a Developing professional**. Rotterdam, The Netherlands: Sense, 2008. v. 4, p. 17-29.

SWARS, S. L., SMITH, S. Z., SMITH, M. E.; HART, L. C. A longitudinal study of effects of a developmental teacher preparation program on elementary prospective teachers' mathematics beliefs. **Journal of Mathematics Teacher Education**, [*S. l.*], v. 12, p. 47–66, 2009.

Processing and editing: Editora Ibero-Americana de Educação.
Review, formatting, standardization, and translation.