

Reflections on the notion of transnumeration: focus on primary school textbooks

Reflexiones sobre la noción de transnumeración: enfoque en los libros de texto para los primeros años de escolaridad

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Abstract

This study aims to analyze in what terms transnumerative thinking is present or potentially present in textbooks for primary school. This qualitative research analysis is based on the theoretical constructs of researchers who support the notion of transnumeration in statistics education. The construction of data was based on the analysis of three collections aimed at the first five grades of elementary school, endorsed by the Programa Nacional do Livro Didático [National Textbook Program]. Some examples of the analyzed textbooks revealed a potential for the development of transnumerative thinking. This analysis concludes that the textbooks present tasks that enable adaptations to develop transnumerative thinking, in line with the indications of the Base Nacional Comum Curricular [National Common Core Curriculum].

Keywords: statistical literacy; transnumeration; textbooks; primary school.

Resumen

Este estudio pretende analizar en qué términos el pensamiento transnumerativo está presente o potencialmente presente en los libros de texto de los primeros años de escolarización. Se trata de una investigación cualitativa cuyos análisis se basan en constructos teóricos de investigadores que sustentan la noción de transnumeración en la educación estadística. La construcción de datos se basó en el análisis de tres colecciones dirigidas a los primeros cinco años de la escuela primaria, avaladas por el Programa Nacional del Libro Didáctico. Se identifica que algunos ejemplos de los libros de texto analizados tienen potencial para el desarrollo del pensamiento transnumerativo. Del análisis de los libros de texto se concluye que presentan tareas que permiten adaptaciones para el desarrollo del pensamiento transnumerativo, en línea con las indicaciones de la Base Nacional Común Curricular.

Palabras clave: literacia estadística; transnumeración; libros de texto; primeros años.

INTRODUCTION

According to Bem-Zvi and Garfield (2004), in an effort to reformulate statistical education to the point of prioritizing the understanding of data and its usefulness for everyday life to the detriment of the mechanical use of statistical tools without a purpose, terms such as statistical literacy, statistical reasoning, and statistical thinking emerged without much understanding about their similarities and differences. Some authors link literacy to learning quantitative skills, while others link reasoning and thinking within a single thread of abilities.

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Statistical literacy is the set of skills needed to understand statistical information and research reports, including knowledge of concepts, vocabulary, symbols, statistical representations, descriptive and inferential tools, and probabilistic concepts.

Statistical reasoning: This process involves how people reason about statistical information and formulate ideas about it; it also reflects on how one statistical concept connects to another, in the search to understand the data. "Reasoning means understanding and being able to explain statistical processes and being able to fully interpret the statistical results" (Bem-Zvi; Garfield, 2004, p. 7, our translation).

Statistical thinking involves using statistical tools to extract information, understand the data, and place it in context. It also allows the subject to understand how statistical investigations occur, why they are important, and what fundamental ideas (the ubiquitous nature of variation, the use of appropriate methods for data analysis, numerical summaries, and visual presentations of the data) are necessary for the investigations to be established.

In our research, we will focus on statistical thinking, in particular on one type of it: transnumerative thinking. We will also seek to understand in which aspect of statistical literacy (Gal, 2002) this type of thinking develops.

According to Snee (1990), statistical thinking is a thought process that recognises the presence of variation around us and in everything we do. It also recognises that all work constitutes a series of interconnected processes that allow us to identify, characterise, quantify, control, and reduce such variation, thus providing opportunities for improvement.

In this sense, Wild and Pfannkuch (1999) point to transnumeration as one of the types of statistical thinking that, in turn, allows the researcher to discover and understand the history of the data. Therefore, we understand that statistical literacy depends on the development of transnumerational skills in students from the first years of schooling, since it is not enough to have the tools: you have to know how to use them and understand their consequences.

This study aims to analyse the terms in which transnumerative thinking is present or potentially present in textbooks in the first years of schooling. To achieve this, we analysed three collections produced for the first five years of elementary school, approved by the National Textbook Program (Programa Nacional do Livro Didático - PNLD) (Brasil, 2019).

TRANSDENUMERATIVE THINKING AS A TOOL FOR STATISTICAL LITERACY

In the field of statistical thinking, our research focuses on transnumerative thinking, which, according to Pfannkuch and Wild (2004), occurs when students handle a series of statistical representations in a meaningful and creative way to look for patterns and communicate ideas critically and in a contextually connected way. In this sense, we highlight what we mean by presenting the *raison d' être* of transnumerative thinking in teacher education in the initial years, the change in the way in which statistical education has historically been approached, the promotion of statistical literacy (and its sophistication) with the development of transnumerative thinking in pedagogy students, seeking, from them, to form a statistical culture that reaches future citizens.

Wild (1994) quotes Bailer (1988), describing a moment in which this author receives letters from councilors expressing their indignation that universities did not teach statistics but mechanical manipulation. She, in turn, reflecting on the possibilities for improving research quality, divided the process into four categories: mental habits, which seek to address knowledge of statistical thinking to connect it with statistical techniques; data collection; data analysis; and communication.

Mental habits belong to dimensions of statistical thinking that are not learned simply through explanations, but are the result of successive practices that, in the course of their applications, constitute a considerable experience for the individual. "We need to inspire habits of mind,

and habits are acquired by doing something over and over again until it becomes automatic” (Wild, 1994, p. 169).

Moore (1990) describes data as numbers in a given context. Following this reasoning, we see the great importance of statistical thinking: any activity involving values of a single variable without context is merely a mechanized calculation and lacks statistical meaning. To mean something, numbers must be closely linked to their context, composing statistical thinking and thus telling their story, describing the world we live in.

Statistical education aims to train statistically literate people capable of thinking about solutions in an interdisciplinary way, mobilising all personal skills. Statistical thinking aims to link these skills with statistical data-processing techniques, giving numbers more context and making them tell their story.

To foster statistical thinking, Burgess (2009) argued for incorporating real research into statistics curricula. However, asking students to do research does not guarantee learning in the area. It is important to develop a theoretical framework so that the experience makes sense and can expand the body of knowledge in the context. Statistical research must have as its ultimate objective in-context learning, since learning is much more than collecting information; it involves synthesising new ideas and information, resulting in a better understanding (Wild; Pfannkuch, 1999).

In composing this theoretical body of statistical research, Wild and Pfannkuch (1999) defined four dimensions of statistical thinking through which the individual researcher can move.

Dimension 01: Investigative cycle. It is about how to act and what is thought during the course of a statistical investigation; Dimension 02: Types of thinking. Five types of thinking inherent in statisticians were listed: recognition of the need for data, transnumeration, variation, a diverse set of models, contextual knowledge, statistical knowledge, and synthesis; Dimension 03: Interrogative cycle. It corresponds to a recursive process of solving statistical problems, in which the thinker is always in one of the interrogative states; Dimension 04: Dispositions. They are related to personal qualities that affect or even initiate a way of thinking.

The focus of our research is on the second dimension of statistical thinking, more specifically, on transnumeration, the second type of thinking, which refers to the alteration and reorganisation of the representations of aspects of the data of a system, to improve its understanding.

Wild and Pfannkuch (1999, p. 227, our translation) define transnumeration:

Transnumeration occurs when we find ways to obtain data (through measurements or classifications) that capture significant elements of the system. It permeates statistical data analysis and occurs whenever we change the way we look at data, in the hope that it will yield new meanings.

In this way, the process of transnumeration involves viewing the data through different graphical or tabular representations, subjecting them to countless statistical models, and transforming and reclassifying them in search of new or other understandings. Moreover, at the end of this process, we will still be using transnumeration when we discover representations of data that help us convey our new understandings of the real system to others, thus forming a dynamic system of changing representations and analyses to generate understanding.

Burgess (2009) defines transnumeration as the ability to identify and select data that appropriately represent the actual situation and, through changes in their representations, reclassifications, or calculations of summary measures, obtain an additional meaning. The author also reinforces the proper use of statistical tools, respecting their characteristics and functionalities, so that the transnumeration process yields reliable insights into the stories contained in the data.

Chick, Pfannkuch and Watson (2005, p. 90) also describe transnumeration as “[...] a type of thinking that captures, creates, defines, and alters measures and representations to seek meaning and learn about observed phenomena.” They add that other attitudes, such as organising, reducing, and summarising data, are part of the perception of the need (or not) for various representations to grasp the history contained in the data and thus the situation in the real world.

Also noteworthy are the observations of Chick, Pfannkuch and Watson (2005), when they point out that the transnumerative tools we will use to understand the data are those that appear in our repertoire of statistical knowledge; that is, the more we acquire statistical content, the greater our chances of transnumeration in the search to understand the stories behind the data.

In addition, Pfannkuch and Rubick (2002, p. 5, our translation) understood this type of statistical thinking as “[...] changing representation to generate understanding”. They established three specific cases of transnumeration when using the statistical system to model the real-world system. These are:

1. i) find measures that reflect qualities and characteristics present in the real system; ii) search for meanings for the data (raw or not), based on their transformations, such as (re)classifications, graphical and/or tabular representations, and statistical summaries; iii) clearly communicate to the target audience the understanding of the meaning of the data in relation to the real system.

In a study with students aged 11 to 13 (which corresponds in Brazil to the 5th to 7th grade of primary education), Pfannkuch, Rubick, and Yoon (2002) managed to describe the students' transnumerative thinking in three main categories: classification and reclassification of data; calculation of averages and construction of graphs; and communication of findings. Other notes made by the authors discuss the need to encourage students to create their own data representations before they are introduced to conventional ones, and to foster creative thinking and encourage innovative representations that promote new insights into the data.

With this, it is worth establishing a framework for our research here, since, based on the studies carried out by Wild and Pfannkuch (1999), Pfannkuch and Rubick (2002), Pfannkuch, Rubick and Yoon (2002) and Chick, Pfannkuch and Watson (2005), and considering the skills described in the Common National Core Curriculum (BNCC) for statistics teaching, we will take as a reference the transnumerative skills to be developed in the early years of elementary school:

2. h₁) classifying and reclassifying data; h₂) constructing charts and tables; h₃) communicating results.

Such skills will be observed at various points throughout this research, always pointing to the central foundation of transnumeration-type statistical thinking highlighted by Wild and Pfannkuch (1999): the search for understanding of the stories contained in the data.

Transnumerative thinking and the practices that foster its development make it possible to reflect on its *raison d'être* in basic education, more precisely in the initial years. In principle, we understand the *raison d'être* of transnumeration at that level as in the words of Pfannkuch, Rubick and Yoon (2002, p. 574, our translation) “[...] Extracting implicit messages from the data requires a constant dialogue among the data, the context, and the student. The thinking of the transnumerative type acts as a catalyst for this conversation.”

Not only the aforementioned authors, but other authors studied and the creators of the term themselves point out, even indirectly, the *raison d'être* of transnumerative thinking as a set of cogs, since they represent the researcher (statistician, researcher, or student), the data investigated and the context that, while making sense, awaits the story or stories revealed by the transnumerated data so that they understand them.

Also noteworthy is the role of statistical tools as the engine of this set, because, as Chick, Pfannkuch and Watson (2005) argue, the more knowledge an individual has about statistical tools, the greater the chances of transnumeration in the search for stories behind the data.

TRANSDUMERATIVE THINKING APPROACH IN RESEARCH IN BRAZIL

To get an overview of research related to the notion of transnumeration, we searched a time range of journal publications from 2015 to 2020, with no period restrictions on doctoral theses and master's dissertations. The databases consulted were the Portal of Periodic Publications of the Coordination for the Improvement of Higher Education Personnel (CAPES) (Brasil, 2024a) and Google Scholar (2024). To search for theses and dissertations, we used the CAPES Theses and Dissertations Bank (Brasil, 2024b) and the Digital Library of Theses and Dissertations (BDTD) (Brasil, 2024c), as they are free sources of access to scientific research in our area.

To search, we used the keywords "*transnumeração*" [transnumeration] and "*pensamento transnumerativo*" [transnumeration thinking] to identify research related to this topic in Brazil. After identifying the relevant works, we read their abstracts to determine whether any study addressed transnumeration or its use as part of its objective, or whether they cited it as part of statistical thinking, as defined by Wild and Pfannkuch (1999). When reading the abstracts of works that discuss transnumeration thinking, we also sought to identify whether the research was linked to teacher training for the first years of elementary school and whether they reported applications of transnumeration at that level of education.

In these searches, we obtained 113 results among articles, undergraduate works, master's dissertations, and doctoral theses that contained the keywords in their content, but most of those works only cited transnumeration or transnumeration thinking as a type of statistical thinking, idealised by Wild and Pfannkuch (1999), that is, they did not address a specific study on the subject.

As for the dissertations and theses that transnumeration raises in some way, we found 14 works: six dissertations and eight theses, of which three of the master's dissertations and seven doctoral theses only mention transnumeration as a type of statistical thinking. We have three master's dissertations and one doctoral thesis in which transnumeration appears as one of the protagonists of the study, contributing to the application and development of the subject in different contexts (Chart 1).

Chart 1. Transnumeration Thinking in Research in Brazil.

Author/year	Title	Level
Giordano (2016)	O desenvolvimento do letramento estatístico por meio de projetos: um estudo com alunos do Ensino Médio (dissertação) [The development of statistical literacy through projects: a study with secondary students] (Master's dissertation)	Master's dissertation
Santos (2017b)	Letramento estatístico nos livros de ensino médio e a Base Nacional Comum Curricular [Statistical literacy in high school textbooks and the Common National Core Curriculum]	Master's dissertation
Santos Júnior (2017)	Letramento estatístico nos livros dos anos finais do ensino fundamental e a Base Nacional Comum Curricular [Statistical literacy in the books for the last grades of elementary school and the Common National Core Curriculum]	Doctoral thesis

Source: the authors.

Chart 1. Continued...

Author/year	Title	Level
Fernandes (2019)	Articulação entre o letramento estatístico de Gal e a compreensão gráfica de Curcio para a formação de professores no âmbito da educação estatística [Articulation between the statistical literacy by Gal and the graphic understanding by Curcio for teacher education in the field of statistical education]	Doctoral thesis

Source: the authors.

The studies indicated in [Chart 1](#) contributed to shaping our perception of statistical literacy and its importance for the formation of citizenship. They also revealed several ways to observe transnumeration as a tool for developing this type of literacy. Take, for example, [Giordano's \(2016\)](#) dissertation, which observed transnumerative praxeologies in high school students' textbooks and notebooks, analysing the specific praxeologies contained in these materials.

The contributions by [Santos \(2017a\)](#) and [Santos Júnior \(2017\)](#) followed the same line as [Giordano's \(2016\)](#) work, since, like the latter, the first two also carried out analysis of textbooks using the concept of praxeologies, according to the anthropological theory of the didactic (ATD), and verified the occurrence or not of tasks that allowed the development of transnumerative thinking. The study of these three dissertations allowed us to understand the importance of beginning to apply tasks that address transnumeration from the initial years, and of providing teachers in this educational period with knowledge of this type of statistical thinking to visualize opportunities for approaching their students.

[Fernandes \(2019\)](#) research allowed us to observe the experience of transnumeration in teacher education. In this thesis, the author discusses transnumeration as a tool for developing statistical literacy, and the importance of this knowledge for teachers of statistics in basic education.

In this study, we found no dissertations or theses produced in Brazil that focused exclusively on the development of transnumeration (transnumerative thinking). We also found that many of the academic productions consulted addressed transnumeration in the field of statistical literacy. Most of them focused their research on the training of mathematics teachers for the last years of primary and secondary education, highlighting the need to deepen their studies in the first years of schooling.

TRANSMERATION AND THE COMMON NATIONAL CORE CURRICULUM

The BNCC brings five thematic units. One of them, Probability and Statistics, is dedicated to stochastic content. As for statistical thinking, the BNCC seeks to encourage research on students' daily lives, encompassing the collection, organization, analysis, and communication of the data obtained, thereby enabling the creation of activities that focus on developing transnumerative thinking skills.

In [Chart 2](#), we present the objects of knowledge and skills described in the BNCC for the thematic unit Probability and Statistics. We were careful to separate the information that only matches the statistical content. We highlight the extracts that indicate the possibility of disseminating transnumerative practices.

Chart 2. Characteristics that enable the development of transnumerative thinking in the BNCC.

Grade	Objects of knowledge	Skills
1st	Reading simple tables and column charts.	(EF01MA21) Reading data expressed in tables and single-column charts.
	Collecting and organizing information. Personal records for the communication of information collected.	(EF01MA22) Conducting research, involving up to two categorical variables of interest and a universe of up to 30 elements, and organizing data through personal representations.
2nd	Collecting, classifying, and representing data in single- and double-entry tables and column charts.	(EF02MA22) Comparing research information presented through double-entry tables and single-column or bar graphs to better understand aspects of the coming reality. (EF02MA23) Conducting research in a universe of up to 30 elements, choosing up to three categorical variables of interest, and organizing the data collected into lists, tables, and graphs of a single column.
	Reading, interpreting, and representing data in double-entry tables and bar charts.	(EF03MA26) Solving problems where data are presented in double-entry tables, bar graphs, or columns. (EF03MA27) Reading, interpreting, and comparing data presented in double-entry tables, bar graphs, or columns that involve significant research results, using terms such as greater and lesser frequency, appropriating this type of language to understand significant aspects of sociocultural reality.
3rd	Collecting, classifying, and representing data regarding categorical variables through tables and charts.	(EF03MA28) Conducting research involving categorical variables in a universe of up to 50 elements, organizing the collected data using lists, single- or double-entry tables, and representing them in single-column charts, with and without the use of digital technologies.
	Reading, interpreting, and representing data in double-entry tables, individual and grouped column charts, bar and column charts, and pictorial charts.	(EF04MA27) Analyzing data presented in single- or double-entry tables and in column or pictorial graphs, based on information from different areas of knowledge, and producing a text summarizing the analysis.
4th	Differentiating between categorical variables and numerical variables.	(EF04MA28) Conducting research involving categorical and numerical variables and organizing the collected data through single or grouped column tables and charts, with and without the use of digital technologies.
	Collecting, classifying, and representing the data of the research carried out.	
5th	Reading, collecting, classifying, interpreting, and representing data in double-entry tables, grouped columnar, pictorial, and linear charts.	(EF05MA24) Interpreting statistical data presented in texts, tables, and charts (columns or lines), referring to other areas of knowledge or other contexts, such as health and traffic, and producing texts to summarize conclusions. (EF05MA25) Conducting research involving categorical and numerical variables, organizing the collected data through tables, columnar, pictorial, and linear charts, with and without the use of digital technologies, and presenting written text about the purpose of the research and the summary of the results.

Source: the authors.

The words data organization and data representation, which are present in the knowledge objects of all initial years, attest to the feasibility of developing transnumerative skills such as h_1 , h_2 , and h_3 . In addition, throughout elementary school, students must themselves develop the skills of preparing, collecting, organizing, and disseminating statistical research.

As knowledge of transnumeration, based on what was suggested by Chick et al. (2005), is an applied statistical practice, to obtain an understanding of the information implicit in the data, the teacher has, in official documents, indications that there are possibilities of developing statistical education practices that reveal the stories contained in them. However, it should be noted that, for this to happen, the teacher must know transnumeration, since the BNCC text does not mention this topic; that is, this knowledge depends on the primary school teacher's initial or continuing education.

In addition to official documents, teachers can also rely on textbooks to guide their classes and sometimes add knowledge. Therefore, it is necessary to study these works to understand whether they make possible, even indirectly, the work of transnumeration.

METHODOLOGY

The research we conducted on the textbooks used was qualitative. We sought to determine whether such textbooks contained tasks that enable teachers from 1st to 5th grade of primary school to develop transnumerative thinking with their students. If this type of task existed in the books, which would make it easier for teachers to approach the subject, the development of this type of statistical thinking would be better founded and within children's reach.

The analysis of the textbooks was based on three collections focused on the first five grades of elementary school, in the version used by the students, which were kindly provided by the Municipal Secretariat of Education of the Municipality of Macapá (SEMED – Macapá). The Ministry of Education selected all the works to compose, together with others, the Mathematics Textbook Guide of the National Textbook Program (PNLD 2019). The works are made available to the teachers of each educational institution, for their free choice and distribution to students.

We also note that all the works analyzed were designed and organized based on the BNCC guidelines. The textbook descriptions are presented in [Chart 3](#). To facilitate discussion and comparison, letters A, B, and C represent each collection, and their volumes are represented by the corresponding elementary school grade; for example, B4 corresponds to Collection B, grade 4.

Chart 3. Collections of math textbooks selected for analysis.

Book	Title	Author(s) (or responsible editor) ¹	Volumes	Editorial	Grade
A	Liga Mundo	Eliane Reame	1st to 5th grades (A ₁ , A ₂ , A ₃ , A ₄ , A ₅)	Saraiva	2017
B	Vem Voar	Júlio Cesar Augustus de Paula Santos	1st to 5th grades (B ₁ , B ₂ , B ₃ , B ₄ , B ₅)	Scipione	2017
C	Ápis	Luiz Roberto Dante	1st to 5th grades (C ₁ , C ₂ , C ₃ , C ₄ , C ₅)	Ática	2017

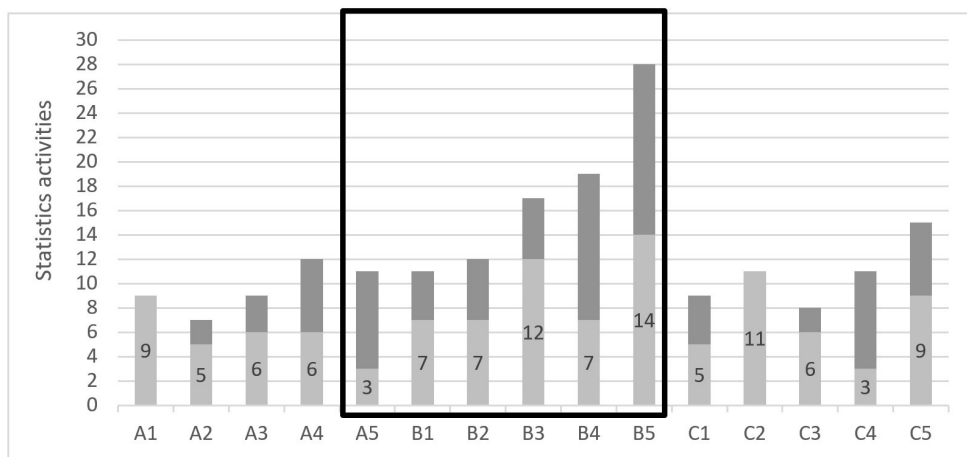
¹Editor responsible for the collections of textbooks produced and organized by the publisher: Reame (2017), Santos (2017a) and Dante (2017a, 2017b, 2017c, 2017d, 2017e). Source: the authors.

RESULTS AND ANALYSIS

In the analysis of the textbook activities, we sought to identify the practices that contributed to the configuration (or reconfiguration) of a relationship between students and the notion

of transnumerative thinking; if so, how they were presented and, if they are negative, once the teacher has a personal relationship with transnumeration, he/she could adapt them to allow his/her students to build said relationship.

In the 15 books, five from each of the three collections, we identified a total of 189 statistical tasks, none of which aimed to develop transnumerative skills. However, in 110 of them, we identified the possibility of additions that would encourage students to seek new understandings and information through data (re)organization, (re)classification, and/or (re)presentation. These tasks could provide moments to tell new stories from the context presented by the statistic (we will call them activities that allow transnumeration) (Figure 1).

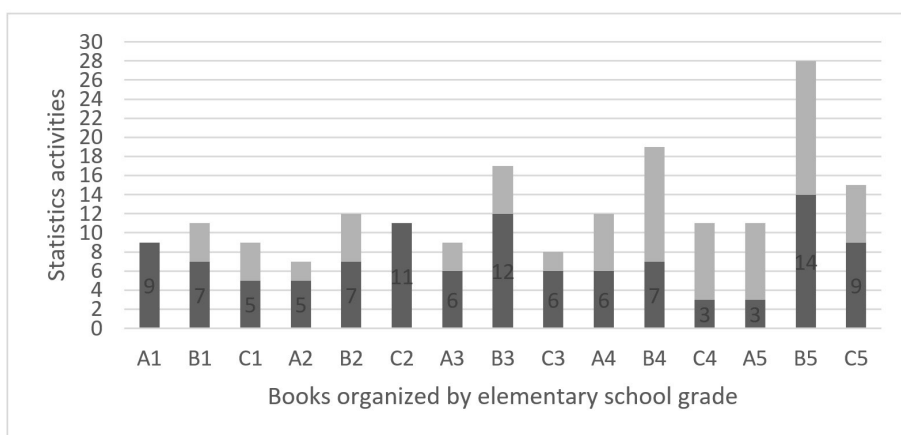


■ Activities that allow transnumeration

Figure 1. Statistical activities that allow transnumeration. Source: the authors.

We also observed that Collection B included a plethora of statistical activities (87 in total), 47 of which included practices that allow teachers to complement them to address transnumeration. In contrast, Collection A had 29 activities with this characteristic, out of a total of 48. As for Collection C, there were 34 out of 54, respectively.

Figure 2 shows a trend that, in our opinion, does not align with the notes of Chick, Pfannkuch and Watson (2005), who argue that the more statistical knowledge we accumulate, the greater the likelihood of developing transnumerative practices to understand the story embedded in the data.



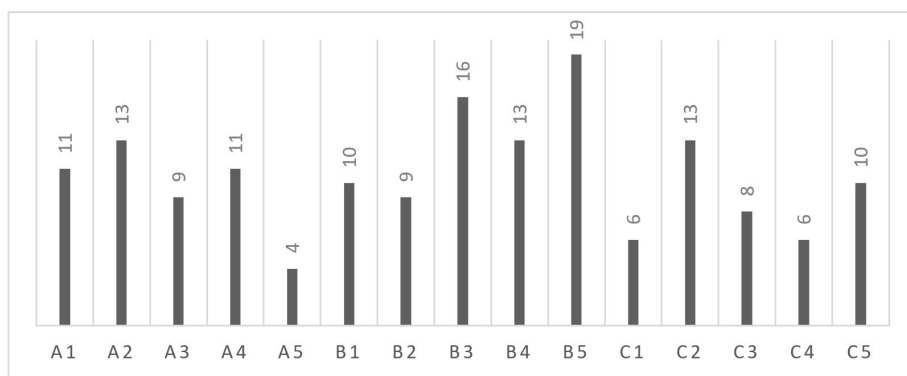
■ Activities that allow transnumeration

Figure 2. The number of statistical activities verified that allow transnumeration, per textbook per grade. Source: the authors.

As the school years progress, activities involving practices that allow transnumeration become less frequent than those that include statistics in the books, which aligns with expectations, as transnumerative tools diversify in the fourth and fifth grades, with the presence of charts of segments and sectors, as well as percentage calculations, which draw on ideas such as Cartesian coordinates, percentages, four operations with numbers, etc.

This fact implies a greater demand on teachers who teach these two stages of primary school, since they cannot rely on the textbook to prepare a practice that can develop transnumerational thinking.

We also analyzed the number of practices that allow transnumeration in each collection (Figure 3). We noted that, in Collections B and C, the number of practices increases as the school years go by, whereas in Collection A, there is a considerable reduction in the fifth grade. It is worth noting that the existence of numerous questions that associate several practices that allow transnumeration helps students realize that the same data can be represented in several different ways, facilitating different perceptions of the same data set.



■ Practices that allow transnumeration.

Figure 3. The quantity of practices that allow transnumeration identified in the textbooks. Source: the authors.

We classified the tasks we identified in the three textbooks that teachers could use to build practices that favor the development of transnumerational skills into eight groups of activities: G_1 : Completing table; G_2 : Building graphs; G_3 : Expressing conclusions; G_4 : Creating list; G_5 : Recording research data; G_6 : Using electronic spreadsheets to process transnumeration; G_7 : Calculating the arithmetic mean; G_8 : Calculating the percentage.

Groups G_1 , G_2 , and G_3 stand out as the most frequent in textbooks, and the only one present in the fifteen volumes was G_1 . This category includes activities such as: (1) completing a table from a list of values; and (2) procedures to count the number of times a value appears in the list presented and subsequently recording this quantity in the indicated table, for each of the values contained in the list (Figure 4).

Of the total copies of the three works, category G_6 only appears in a single activity in A_4 , which leads us to reflect on the lack of incentive to use electronic spreadsheets in public schools, given that, at least in theory, they have computer classrooms to carry out practices of this type and other subjects, in addition to the use of the computer as a tool for digital inclusion (Figure 5). In other words, the main statistical tool is not presented to students in the initial years. We believe that one way to encourage the use of this tool and to draw managers' attention to computer classrooms is to include more tasks of this type in all textbooks.

Coutinho (2017) highlights the importance of developing practices that favor the use of technological tools to enhance statistical thinking, so that concepts can be built rather than merely setting mechanized procedures. We should also remember that the BNCC recommends

2 João resolveu refazer a tabela da atividade 1 de um jeito diferente. Em vez de escrever a pontuação obtida, ele anotou a quantidade de latas que cada um deles derrubou durante a partida.

- a) Observe na tabela da página 84 quantos pontos cada jogador fez e complete a tabela abaixo com a quantidade de latas derrubadas.



Latas derrubadas por jogador

Jogador	1ª rodada	2ª rodada	3ª rodada	Total de latas
João	6	10	8	24
Paulo				
Priscila				
Maria				

Dados fictícios.

- b) Agora, registre no gráfico abaixo a quantidade total de latas que cada jogador derrubou nessa partida. Cada quadradinho deve representar 1 lata derrubada. Use uma cor diferente para cada jogador.

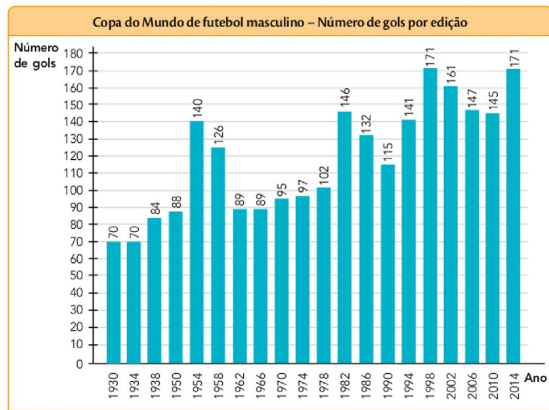


Dados fictícios.

85

Figure 4. Activity of completing a table. Source: B₃ (p. 85).

- 2 Agora, observe a quantidade de gols por edição da Copa do Mundo.



Fonte dos dados: Fédération Internationale de Football Association (FIFA). FIFA Tournaments. Disponível em: <www.fifa.com/fifa-tournaments/statistics-and-records/worldcup/index.html>. Acesso em: 15 fev. 2017.

Responda às questões a seguir.

- a) Quantos continentes já sediaram Copas do Mundo de futebol masculino?
Quais são eles? _____
- b) Qual é o continente que sediou o maior número de Copas do Mundo de futebol masculino? _____
- c) Observando o mapa e o gráfico do número de gols, quantos gols já foram marcados em cada continente que sediou a Copa do Mundo de futebol masculino?

💡 Crie em seu caderno uma tabela a partir dos dados do gráfico de 2002 a 2014. Em seguida, pesquise a quantidade de gols marcados na Copa de 2018 realizada na Rússia e complete a tabela com esses dados. Você pode também reproduzir essa tabela em um programa de planilhas eletrônicas e criar um gráfico.

sessenta e cinco

65

Figure 5. Activity of use of electronic spreadsheets. Source: A₄.

using calculators and spreadsheets to compare results, build graphs, and calculate measures of central tendency.

As for categories G_7 and G_8 , they are only explored in B_5 and C_5 copies, respectively, because, at this grade, students already have sufficient mathematical skills to begin with the concepts of arithmetic mean and percentage. However, Collection A did not introduce the content (Figure 6).

Estadística: média de 2 ou mais números

Explorar e Descobrir

ATIVIDADE EM GRUPO (TODA A TURMA)

Parte 1

- Formem 4 pilhas de livros, uma com 2 livros, uma com 5 livros, outra com 2 livros e a última com 3 livros, como nesta imagem.
- Façam uma arrumação de modo que os livros fiquem ainda em 4 pilhas, mas todas com o mesmo número de livros.
- Agora, respondam (cada um em seu livro).
 - Quantos livros havia em cada pilha antes da arrumação?
 - Quantos livros ficaram em cada pilha depois da arrumação?
 Veja como podemos ilustrar essa situação.



Parte 2

- Agora vamos pensar em quais operações matemáticas devem ser efetuadas para determinar o número total de livros e o número de livros em cada pilha, depois da arrumação. Respondam (cada um em seu livro).
 - Quais operações matemáticas foram realizadas nessa situação?
 - Use os dados do problema para indicar e efetuar essas operações.
 - Completem.

Dizemos que o número _____ é a **média** dos números _____.

- 7 Durante 1 mês do ano os alunos das 3 turmas do 5º ano da escola de Augusto retiraram da biblioteca 300 livros ao todo. Os alunos do 5º ano **A** retiraram 40% do total. Os alunos do 5º ano **B** retiraram 25% do total. Calcule e complete a tabela.

Livros retirados da biblioteca

Turma	Número de livros retirados	Porcentagem do total
5º A		40%
5º B		25%
5º C		

Tabela elaborada para fins didáticos.

- 8 Uma roleta tem 5 setores de tamanhos iguais marcados com os números de 1 a 5. Essa roleta será girada. Registre a probabilidade em cada caso.
- De sair um número ímpar.
 - De 20 ser um múltiplo do número que sair.



9 NÚMEROS CRUZADOS

Calcule e use os resultados das horizontais para preencher o quadro (1 algarismo em cada quadrinho). Depois, calcule e use os resultados das verticais para conferir.

	D	E	F	Horizontais	Verticais
A →				A: 50% de 224 = _____	D: 10% de 1310 = _____
B →				B: 25% de 1200 = _____	E: $\frac{3}{2}$ de 70 = _____
C →				C: $\frac{1}{6}$ de 900 = _____	F: 20% de 1000 = _____

Figure 6. Activity to get started on the concepts of the arithmetic mean and percentage. Source: C_5 (p. 93; p. 159).

We were struck by the G_3 -type task, which appeared only in B3 (Figure 7), in which the author leads the student to reflect on how to represent data to facilitate their understanding. In these tasks, students must determine the best way to represent the data for a situation, analyze multiple ways to represent it, and conclude which form best facilitates their perception of the information about the situation.

When analyzing the following task:

f) In your opinion, is it easier to see who dropped more cans in the chart or in the table on page 85? (See Figure 4) Explain to your peers and the teacher (B₃, p. 86).

We realize that there are two activities: concluding the best way to represent the data for a situation, and explaining the conclusion to peers and the teacher. Regarding G_3 , students are led to reflect not only on the various ways of representing the data but also on the importance of having this diversity. Each presents a way of seeing and understanding information. In this case, we understand that transnumeration occurs because, when reflecting on the two representations and deciding on one, the student perceives the information in the data of each representation.

We identified that the tasks in the textbooks have the potential to develop transnumerative thinking, as long as the teacher favors practices that allow transnumeration, such as assigning tasks that prompt discussion of the stories in the data.

- 3 Consulte o gráfico da página 85 e responda às questões abaixo.
- Quantas latas Maria derrubou a mais do que Paulo? _____
 - Quantas latas a mais o jogador que ficou em último lugar precisava derrubar para alcançar o vencedor? _____
 - Essa quantidade de latas corresponde a quantos pontos? _____
 - Quantas latas foram derrubadas nessa partida? _____
 - Essa quantidade de latas corresponde a quantos pontos? _____
- f) Na sua opinião, é mais fácil visualizar quem derrubou mais latas no gráfico ou na tabela na página 85? Explique aos colegas e ao professor.
- 4 Agora vamos fazer o gráfico da partida do jogo **Tomba-latas** da página 74 que você e seus colegas jogaram.
- Primeiro, consulte a tabela que você preencheu na página 74. Depois preencha a tabela abaixo com a quantidade de latas derrubadas durante a partida.

Latas derrubadas por jogador

Jogador	1ª rodada	2ª rodada	3ª rodada	Total de latas

Dados do jogo **Tomba-latas**.

86

Figure 7. Activity of reflecting on how to represent data that facilitates their understanding. Source: B₃ (p. 86).

CONCLUSION

In our research, we showed that the textbooks analyzed do not emphasize discussions about basic elements of statistical education, such as the perception of data variation, transnumeration, etc. On the contrary, they are dedicated to exploring mathematical calculations and reading the relationships between the axes of a graph.

Although there are no managerial tasks that allow the development of transnumerational skills, in the first grades of elementary school, both official documents and textbooks offer possibilities for the constitution of these practices. However, this will depend on the teachers' management. Therefore, they must have contact with the notion of transnumerative thinking.

In general terms, we conclude that the tasks included in the textbooks allow us to say that, although we note that some can be adapted for the development of transnumerative thinking, in line with the BNCC, if the teacher who teaches primary school does not know statistical literacy—in particular, transnumerative thinking and how it evolves—, they will not be able to adapt and complement the tasks for students to develop these skills.

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