DEVELOPING CREATIVE ACTIVITY ABILITIES OF STUDENTS IN HIGHER EDUCATIONAL ESTABLISHMENTS

DESENVOLVIMENTO DE HABILIDADES DE ATIVIDADE CRIATIVA DE ESTUDANTES EM ESTABELECIMENTOS DE ENSINO SUPERIOR

DESARROLLO DE LAS HABILIDADES DE ACTIVIDAD CREATIVA DE ESTUDIANTES EN ESTABLECIMIENTOS EDUCATIVOS SUPERIORES

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ABSTRACT: The article discusses methods of solving geometric problems with the active use of methods such as analysis and synthesis, analogy and generalization, based on theoretical thinking about the principle of rising from simple to complex, to develop students' capacity for the creative activity. The authors developed problem systems, focused on building the capacity to "make" independent discoveries both in the process of solving a problem and in the phase of research for the result. The problem system developed aims to find a way to solve a more complex problem, after a similar method been used in another simpler or more particular problem. Participants in the experiment are future masters in pedagogical education (profile "Mathematics Education") at Togliatti State University. The article shows that the most effective methods to prepare future masters in mathematics education for creative professional activity can be methods such as scientific knowledge as analogy and generalization. It was found that in the learning process of solving geometric problems inserted in the developed system, students present superior indicators of the level of formation of creative activity, as a result of the development of the capacity of the future teacher of pedagogical education (profile "Mathematics Education") to analogy and its application in specific situations, its ability to use established properties, formed skills and abilities, techniques and methods of action in relation to another object under new conditions and for new purposes, the use of mathematical concepts and theorems in specific problems increasingly diverse.

KEYWORDS: Geometry. Task. Analogy. Generalization. Creative activity.

RESUMO*: O artigo discute métodos de resolução de problemas geométricos com o uso ativo de métodos como análise e síntese, analogia e generalização, com base no pensamento teórico sobre o princípio da ascensão do simples ao complexo, a fim de desenvolver a*

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capacidade dos alunos para a atividade criativa. Os autores desenvolveram sistemas de problemas, focados na formação da capacidade de "fazer" descobertas independentes tanto no processo de resolução de um problema quanto na fase de pesquisa do resultado da solução. O sistema de problemas desenvolvido visa encontrar uma maneira de resolver um problema mais complexo após um método semelhante ter sido usado em relação a outro problema mais simples ou particular. Os participantes do experimento são futuros mestres em educação pedagógica (perfil "Educação Matemática") na Universidade Estadual Togliatti. O artigo mostra que os métodos mais eficazes de preparar futuros mestres em educação matemática para a atividade profissional criativa podem ser métodos em que o conhecimento científico é tomado como analogia e generalização. Verificou-se que no processo de aprendizagem da resolução de problemas geométricos inseridos no sistema desenvolvido, os alunos apresentam indicadores superiores no nível de formação da atividade criativa, em resultado do desenvolvimento da capacidade do futuro mestre de educação pedagógica (perfil "Educação Matemática") à analogia e sua aplicação em situações específicas, sua capacidade de usar as propriedades estabelecidas, habilidades e capacidades formadas, técnicas e métodos de ação em relação a outro objeto em novas condições e para novos fins, o uso de conceitos matemáticos e teoremas em problemas específicos cada vez mais diversos.

PALAVRAS-CHAVE: *Geometria. Tarefa. Analogia. Generalização. Atividade criativa.*

RESUMEN*: El artículo discute métodos para la resolución de problemas geométricos con el uso activo de métodos como análisis y síntesis, analogía y generalización, basados en el pensamiento teórico sobre el principio de ascenso de simple a complejo para desarrollar la capacidad de los estudiantes para la actividad creativa. Los autores han desarrollado sistemas de problemas, enfocados en la formación de su capacidad para "hacer" descubrimientos independientes tanto en el proceso de resolución de un problema como en la etapa de investigación del resultado de la solución. El sistema de problemas desarrollado tiene como objetivo encontrar una manera de resolver un problema más complejo, después de que se haya utilizado un método similar en relación con otro problema más simple o particular. Los participantes en el experimento son futuros maestros de la educación pedagógica (perfil "Educación Matemática") en la Universidad Estatal de Togliatti. El artículo muestra que los métodos más efectivos para preparar a los futuros maestros de la educación matemática para la actividad profesional creativa pueden ser métodos de conocimiento científico como la analogía y la generalización. Se reveló que en el proceso de aprendizaje para resolver problemas geométricos incluidos en el sistema desarrollado, los estudiantes demuestran indicadores más altos del nivel de formación de la actividad creativa, como resultado del desarrollo de la capacidad del futuro maestro de la educación pedagógica (perfil "Educación Matemática") a la analogía y su aplicación en situaciones específicas, su capacidad para utilizar las propiedades establecidas, destrezas y habilidades formadas, técnicas y métodos de acción en relación con otro objeto en nuevas condiciones y para nuevos propósitos, el uso de conceptos matemáticos y teoremas en problemas específicos cada vez más diversos*

PALABRAS CLAVE*: Geometría. Tarea. Analogía. Generalización. Actividad creativa.*

Introduction

Theoretical research and generalization of our own pedagogical experience and pedagogical experience of famous teachers and teachers of mathematics (DOROFEEV *et al.,* 2018; GLAZKOV; EGUPOVA, 2017; KALINKINA, 1995; KALMYKOVA, 2013) led us to understand that the modern master pedagogical education (profile "Mathematical Education") must possess at a sufficiently high level both the mathematical apparatus and the methods of teaching mathematical disciplines, be able to show their trust in students, act as a source of human experience accumulated throughout the entire time of human existence on earth, which can use to enrich their knowledge and understanding of the environment; to feel the emotional mood of each student, to be able to openly express, accept and understand their mental state and their experiences, should be able to show their creative resourcefulness in the process of teaching mathematical methods of cognition of the surrounding world (DOROFEEV, 2000; VYGOTSKY, 2007; TEMERBEKOVA *et al.,* 2013; VAGANOVA *et al.,* 2020). The concept of creative activity is quite complex and multifaceted, the meaning and content of this concept is constantly being refined, replenished and improved. The levels of manifestation of creative activity by students depend on many internal and external factors, both dependent on them and independent, the level of ontogenetic development of everyone, his psychophysiological and mental state, individual psychological characteristics, the level of upbringing and preparedness to perceive a particular mathematical fact. or a concept, the level of development of his intellectual abilities, etc. (KALINKINA, 1995; KALMYKOVA, 2013).

Psychological science provides our attention with many different approaches to the interpretation of the concepts of "creative activity" and "creative activity". As a rule, creative activity is associated with the manifestation of the active nature of creative activity in certain forms. From a psychological point of view, creative activity can be interpreted either as a set of properties of the human nervous system, or as a certain mental state of a person, or as a characteristic of a person's vital activity, or as its property. Thus, it can be argued that creative activity is determined by the action of both internal and external factors, each of which is based on the most important thing - the desire and ability of the student to discover new facts and new knowledge previously unknown to him (DAVYDOV, 2000; PODLASY, 2001; WINTER, 2006).

In the process of the formation of creative activity, all mental processes are involved simultaneously or in a certain sequence, conditioned by our sensations, perception, attention, imagination, emotions, memory, thinking. When they interact, objects of the real world are reflected and their images are formed in our consciousness, personal perception of reality at a given moment in time and in each situation. Modern pedagogical science knows various means, techniques, methods and forms that contribute to the development of creative activity. However, until now we do not know to what extent, in what conditions and when it is possible to use this or that teaching method, this or that form of organization of educational and cognitive activity, this or that means of teaching, in order to say with confidence that the chosen by us in a certain system means, methods and forms with great efficiency contribute to the formation of creative activity (ANDREEV, 1988; DAVYDOV, 2000; LERNER, 2016; MUDRIK, 2004).

Methodology

The development of students' creative activity largely depends on teaching mathematical concepts and methods of the type of scientific knowledge with the active use of methods such as analysis and synthesis, analogy and generalization, concretization and comparison, based on theoretical thinking on the principle of ascent from simple to complex, from the abstract to the concrete, from the particular to the general, using advanced learning technologies, for example, student-centered learning, differentiated learning, learning through UDE, computer and digital technologies (DAVYDOV, 2000; MUDRIK, 2004; UTEEVA, 2015; VAGANOVA *et al.,* 2020).

In our concept of the formation of creative activity in future masters of pedagogical education, we adhere to a personal approach and a humanistic orientation in preparing them for the organization of creative activity. The ability of the future Master of Pedagogical Education (profile "Mathematical Education") to analogy and its application in specific situations is characterized by his ability to use established properties, skills and abilities formed, techniques and methods of action in relation to another object in new conditions and for new purposes. The development of the ability to analogy is facilitated by the process of using mathematical concepts and theorems in more and more diverse specific problems (DAVYDOV, 2000; DOROFEEV, 2000; LODATKO, 2015; MENCHINSKAYA, 2004).

These can be problems of finding a way to solve a more complex problem, after a similar method has been used in relation to another simpler or particular problem. The ability of the future master of pedagogical education (profile "Mathematical Education") to abstraction and its application in specific situations is characterized by his ability to highlight certain features in the object under study. The development of the ability to abstraction is facilitated by the teacher's ability to lead his pupils to each new concept, to each new theorem with the help of well-chosen examples for comparison, highlighting common features or general regular connections between features and formulating the necessary conclusion by the students themselves. This approach to the introduction of new concepts and previously unknown facts to students develops the ability not only to abstraction, but also to generalize. The ability of the future teacher of mathematics to generalize and apply it in specific situations is characterized by the ability to identify common features in a number of objects and group objects on this basis. The wider and more diverse the generalizations, the more independence the students themselves show, the more effective is the effect of the method of summing up a concept with the help of examples on the formation of their creative activity. It is recommended to move from generalizations, which are based on specific examples and lead to particular conclusions, to generalizations, which are based on several concepts and facts, gradually expanding the circle of generalized material (DAVYDOV, 2000; LODATKO, 2015; SAMYGIN; STOLYARENKO, 2012; UTEEVA; ORAZYMBETOVA, 2012).

Results

The formation of creative activity in the future master of pedagogical education (profile "Mathematical education") is inextricably linked with the formation of the ability to compose a whole from its parts and break the whole into its constituent parts. As you know, students get acquainted with the first ideas about the category of the whole in basic school when studying fractions. There they learn to break a whole, for example, an apple into its constituent parts, highlighting one second or one third, etc. Later, throughout the entire period of study, the mathematical ability to divide the whole into parts and make up a whole from its parts is gradually transformed into the philosophical category of the whole. In mathematics, the category of the whole is understood as the completeness of the solution of the problem, the completeness of the system of axioms, or the closed nature of the mathematical process. For example, when solving irrational equations, one of the most common ways is to raise both sides of the equation to the desired power. It is clear that in this case, it is possible that extra roots will be acquired. Restricting the solution to an irrational equation by the roots of the newly obtained equation generates an incompleteness of the problem posed. In this task, only part of it is completed: the original equation is replaced by a more general one - algebraic, obtained from the given by freeing from radicals. To ensure the integrity of the solution to an irrational equation, it is necessary to find out which of the roots of the algebraic equation are the roots of the irrational equation, and which are not, i.e., go beyond the scope of definition. The standard replacement of an irrational equation with an algebraic one obtained from a given one by raising both parts to the appropriate power entails a violation of one of the basic laws of dialectics: the law of negation of negation. According to this law, the old is not simply discarded and replaced by the new, but in accordance with the principle of continuity, from the former is taken what is necessary for the development of the new. In this case, we do not just replace the irrational equation with an algebraic one, but take into account that the original equation contains radicals that impose certain restrictions on unknown quantities.

Let us explain this with a specific example: Find the largest root of the equation:

$\sqrt{x^2+4x+7}=\sqrt{13-x}$. To get an unambiguous answer to the question posed, it is necessary to resolve a specific problem situation. To this end, we will square both sides of the equation. As a result, we obtain a new equation $x^2+4x+7=13-x$ It should be noted that the domain of the first equation is narrower than the domain of the second equation. The domain of the first equation is the interval $(-\infty;\ \ 13]$, the domain of the second is the entire number line. The domain of the initially given equation is the interval: $(-\infty;\ \ 13]$. Hence, from the roots of the quadratic equation $x^2+5x-6=0$ we need to select those that fall within this interval. Solving the second equation, we find that $x_1\ =\ 1,\ x_2\ =\ -6$. Both numbers fall into the domain of the first equation, which means that they serve as the roots of the equation. Choosing the largest – 1. Ans.: 1.

In the practice of teaching schoolchildren to solve irrational equations, quite often there are those, the scope of which narrows down to the roots of this equation. In this regard, consider the equation: $\sqrt{-x^2+8x-15}=\sqrt{x-5}$. Using the above technique, we will square both sides of the equation. As a result, we obtain the algebraic equation $-x^2+8x-15=x-5$. After elementary transformations, we bring this equation to the form $x^2-7x+10=0$. Solving this equation, we find that $x_1\ =\ 2,\ x_2\ =\ 5$. Now we will find the domain of definition of the given equation. We demand that $\begin{cases}-x^2+8x-15\geq 0,\\ x-5\geq 0\end{cases}$.

Solving this system, we find that $x\ =\ 5$. This means that the domain of definition of this irrational equation consists of only one number 5, which is the root of the equation. Thus, to solve irrational equations, it is advisable to first find the domain of its definition, and then use the methods of getting rid of roots. Because sometimes the roots are easily obtained in the process of finding the domain of definition of an irrational equation.

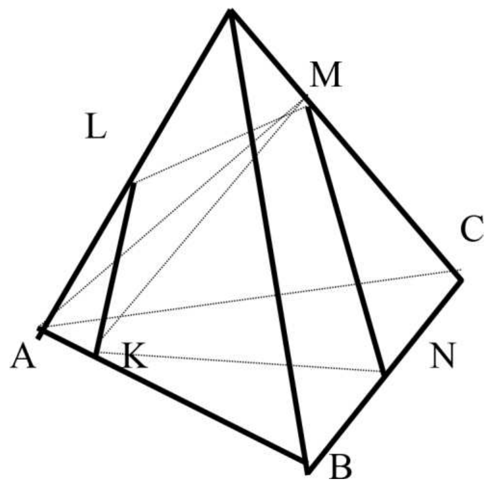
In the methods of teaching geometry, a fusionist approach to teaching students geometric facts and concepts has long been known, which involves the joint study of planimetric and stereometric figures. The effectiveness of this approach lies in the fact that it largely contributes to the formation of students' ability to independently, on the basis of previously learned material, discover new knowledge. Let us illustrate this by an example of studying the properties of the medians of a triangle and a tetrahedron. As you know, the medians of a triangle intersect at one point and divide it in a ratio of 2: 1, counting from the vertices, the medians of a tetrahedron also intersect at one point and divide it in a ratio of 3: 1, counting from the vertices. In classical textbooks on geometry for grades 7-9 and grades 10-11, it is assumed that these facts are directly communicated to students, which they remember, sometimes unconsciously. Hence the problems in the application of these facts when solving geometric problems of both planimetric and stereometric nature.

The schoolboy did not make much effort in assimilating these facts. His consciousness did not show the proper mental activity for this fact to go through certain stages of its assimilation. In order to stimulate the mental activity of students, some teachers actively use visual teaching methods, for example, an image of an arbitrary triangle is built on a computer screen and the midpoints of its sides are marked, then its medians are drawn. As the medians are plotted, the students notice that the two medians intersect at a point, it turns out that the third median of the triangle also passed through this point. Further, the teacher proposes to change the shape of the triangle to another and see if the median of this triangle has this property? Students find with interest that the medians of this triangle also intersect at one point. Thus, with the help of simple examples, we have led our schoolchildren to put forward a hypothetical idea that the medians of an arbitrary triangle intersect at one point. Now it is important for the teacher to maintain and increase this interest in order to move on to proving this fact. It should be strict, but not dry.

Each student must penetrate the idea of proving this fact in order to understand its content deeper and broader. In the process of proving this theorem, students discover another important fact that is new to them, that the medians of a triangle are divided by their point of intersection in the ratio 2: 1, counting from the vertex. With the aim of a more conscious perception of the theorem, students can be offered to solve the following planimetric problems: In a regular triangle ABC medians AA_1, BB_1, CC_1, intersecting at a point O. Prove that triangle is equal to triangle. Can it be argued that triangles OBA_1, OCA_1, OAB_1, OAC_1 are equal. If to paint a triangle OCB_1 23 grams of yellow paint is required, how much of such paint is required to paint the entire triangle? This stage of assimilation of any mathematical fact is necessary, if only because in the process of teaching schoolchildren to prove mathematical theorems, an important idea is formed that it is not enough to find some pattern, even in real life, it is also necessary to substantiate its right to exist and apply in activities. As we have already noted, the spatial analogue of a triangle is a tetrahedron - a polyhedron defined by four points that do not lie in the same plane, and four triangles with vertices at these points. Each triangle has a center of gravity - the point of intersection of its medians.

The median of a tetrahedron is a line segment connecting its vertex with the center of gravity of the opposite face. Using the capabilities of computer technologies, it is possible to visually convince schoolchildren of the hypothetical idea that the medians of a tetrahedron intersect at one point, which they can put forward in the process of getting acquainted with such a concept as the median of a tetrahedron. In school textbooks on geometry for grades 10-11, this fact is not studied as a separate theorem, but it can be proposed as a separate problem, for example, when studying the basics of the vector-coordinate method. In the process of solving this problem, schoolchildren will come to the discovery of a new stronger result that the medians of a tetrahedron not only intersect at one point, but moreover divide it in a ratio of 3: 1, counting from the vertices.

For a deeper understanding of this fact and the formation of students' ability to make discoveries, to reveal hidden facts both in the task itself and in the course of its solution, the following task will contribute: The tetrahedron ABCD is given. Points K and M are taken on its edges AB and CD so that AК:KB = DM:MC ≠ 1. A plane is drawn through points K and M, dividing the tetrahedron into two polyhedrons of equal volumes. In what respect does this plane divide the BC edge?



The polyhedron located under the MLNK cutting plane can be divided into three tetrahedra AMKN, AMCN, ALMK. The volume of each tetrahedron can be represented as a sixth part of the module of mixed products $\mathrm{mod}(\overrightarrow{AM},\overrightarrow{AN},\overrightarrow{AK})$, $\mathrm{mod}(\overrightarrow{AM},\overrightarrow{AC},\overrightarrow{AN})$, $\mathrm{mod}(\overrightarrow{AL},\overrightarrow{AM},\overrightarrow{AK})$. If we put that $(AB,\ K) = (DC,\ M) = \lambda$, $(AB,\ K) = (DC,\ M) = x$, то

$$\overrightarrow{KB} = \frac{1}{1+\lambda}\overrightarrow{AB},\quad \overrightarrow{AK} = \frac{\lambda}{1+\lambda}\overrightarrow{AB},\quad \overrightarrow{MC} = \frac{1}{1+\lambda}(\overrightarrow{AC}-\overrightarrow{AD}),$$

$$\overrightarrow{LA} = \frac{1}{1+x}\overrightarrow{DA},\quad \overrightarrow{DL} = \frac{x}{1+x}\overrightarrow{DA},\quad \overrightarrow{DM} = \frac{\lambda}{1+\lambda}(\overrightarrow{AC}-\overrightarrow{AD}),$$

$$\overrightarrow{NB} = \frac{1}{1+x}(\overrightarrow{AB}-\overrightarrow{AC}),\quad \overrightarrow{CN} = \frac{x}{1+x}(\overrightarrow{AB}-\overrightarrow{AC}).$$

Considering these relations and the properties of the mixed product, we obtain that

$$\mathrm{mod}(\overrightarrow{AM},\overrightarrow{AK},\overrightarrow{AN}) = \frac{\lambda}{(1+\lambda)^2(1+x)}\mathrm{mod}(\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{AD}),$$

$$\mathrm{mod}(\overrightarrow{AM},\overrightarrow{AN},\overrightarrow{AC}) = \frac{x}{(1+\lambda)(1+x)}\mathrm{mod}(\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{AD}),$$

$$\mathrm{mod}(\overrightarrow{AL},\overrightarrow{AM},\overrightarrow{AK}) = \frac{\lambda^2}{(1+\lambda)^2(1+x)}\mathrm{mod}(\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{AD}).$$

As

$$0.5\,\mathrm{mod}(\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{AD}) = \mathrm{mod}(\overrightarrow{AM},\overrightarrow{AN},\overrightarrow{AK}) + \mathrm{mod}(\overrightarrow{AM},\overrightarrow{AN},\overrightarrow{AC}) + \mathrm{mod}(\overrightarrow{AL},\overrightarrow{AM},\overrightarrow{AK}),$$

then $x=1$. It means, that the plane KLMN not only intersects the BC edge but divides it in half.

In terms of the formation of the ability to make new discoveries and apply the mixed product of vectors to solving school geometric problems and the ability to compose new problems, the following mathematical exercise acquires: A tetrahedron ABCD is given, the volume of which is 1. Plane α crosses the ribs DA, DB, AC, CB respectively at points K, L, P, M such that $DK = 2KA, LB = 2DL, CM = 3MB$. Find the volume of the pyramid. Make up possible generalizations of the problem.

The scope of the search for a solution to the problem must be narrowed by focusing the attention of trainees on the possibility of splitting a quadrangular pyramid into two tetrahedra and applying the module of the mixed product of vectors to calculating the volumes of the resulting tetrahedra. The ability to break a figure into its constituent parts contributes to the formation of the ability to divide the whole into parts; use analysis when looking for a solution to a specific problem. In this case, the whole is a quadrangular pyramid LABMP, and its parts are two tetrahedra ABLP and BMPL. Actualization of the expedient application of the module of the mixed product of vectors to the calculation of the volumes of tetrahedrons contributes to the formation of the ability to apply the mixed product to solving school geometric problems. The ability of trainees to apply a mixed product of vectors to the calculation of the volumes of tetrahedra allows one to obtain a number of useful relations: $V_1 = \left|(\overrightarrow{AL}, \overrightarrow{AB}, \overrightarrow{AP})\right|/6$, $V_2 = \left|(\overrightarrow{BM}, \overrightarrow{BP}, \overrightarrow{BL})\right|/6$, $V_{DABC} = \left|(\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD})\right|/6$. As $V_{DABC} = 1$, то $\left|(\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD})\right| = 6$.

The implementation of such logical reasoning generates among students the desire to express mixed products of vectors $\overrightarrow{AL}, \overrightarrow{AB}, \overrightarrow{AP}$; $\overrightarrow{BM}, \overrightarrow{BP}, \overrightarrow{BL}$ through the mixed product of basis vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$. Expressions of the mixed product of one triplet of vectors through the mixed product of another depends on the ability of the trainees to represent some vectors as a linear combination of others; apply properties of the mixed product to its calculation. An important didactic significance of this task also lies in the fact that at the stage of calculating the coefficient α collinear vectors $\overrightarrow{AP}$ и $\overrightarrow{AC}$ the trainees are improving their ability to apply the necessary and sufficient condition for vectors coplanarity in specific situations. Using the condition of the problem and the properties of the tetrahedron, one can show that

$$\overrightarrow{AL} = \frac{\overrightarrow{AB} + 2\overrightarrow{AD}}{3}, \quad \overrightarrow{BL} = \frac{2(\overrightarrow{AD} - \overrightarrow{AB})}{3}, \quad \overrightarrow{BM} = \frac{\overrightarrow{AC} - \overrightarrow{AB}}{4}.$$

To determine the coefficient α collinear vectors $\overrightarrow{AP}$ и $\overrightarrow{AC}$ you can use the fact that the mixed work $(\overrightarrow{PL}, \overrightarrow{PM}, \overrightarrow{LM})$ of vectors $\overrightarrow{PL}, \overrightarrow{PM}, \overrightarrow{LM}$ equals 0. As

$$\overrightarrow{PL} = \frac{\overrightarrow{AB} + 2\overrightarrow{AD} - 3\alpha\overrightarrow{AC}}{3},$$

$$\overrightarrow{PM} = \frac{3\overrightarrow{AB} + (1 - 4\alpha)\overrightarrow{AC}}{4}, \quad \overrightarrow{LM} = \frac{15\overrightarrow{AB} + 3\overrightarrow{AC} - 8\overrightarrow{AD}}{12},$$

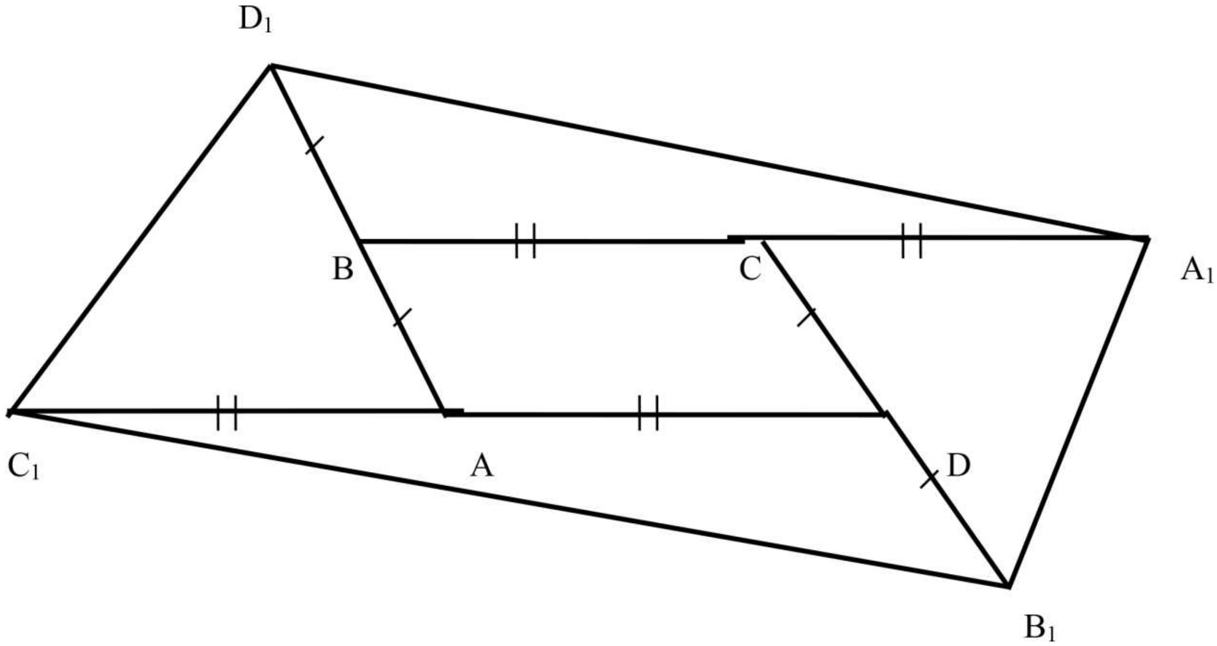
then the mixed product of vectors ($\overrightarrow{PL}, \overrightarrow{PM}, \overrightarrow{LM}$) equals $\frac{5-20\alpha}{72}$. Hence $\alpha = \frac{1}{4}$. So we get that $\overrightarrow{AP} = \overrightarrow{AC}/4$, $\overrightarrow{BP} = \overrightarrow{AB} - \overrightarrow{AC}/4$.

Using the properties of the mixed product of vectors, we find the volume of the tetrahedron ABLP (1/6) and the volume of the tetrahedron BMPL (1/8), and then, by connecting the parts into a single whole, we find the volume of the quadrangular pyramid LABMP.

In order to form a more conscious perception of the mixed product of vectors and develop students' ability to apply the properties of the mixed product of vectors to solving geometric problems, it is advisable to consider tasks of the following type:

On the rays AB, BC, CA, containing the corresponding sides of the triangle ABC, points taken C_1, B_1, A_1, such that $AC_1 = AB$, $CA_1 = BC$, $AB_1 = CA$. Find the ratio of the area of a triangle ABC to the area of the triangle $A_1B_1C_1$.

First of all, it should be noted that the quantities and relationships between them given in this problem are affine-invariant. At this stage of solving the problem, future mathematics teachers develop the ability to select affine-invariant objects. The presence of such objects in a problem allows it to be specialized, which can serve as the basis for finding its optimal solution. Concretization of this problem leads to a new one, which is obtained from the previous replacement of an arbitrary triangle with an equilateral one. If the triangle $A'B'C'$ regular, then triangle $A_1'B_1'C_1'$ is also regular, and it is composed of three equal triangles $A'B'C_1'$, $C'B'A_1'$, $A'C'B_1'$ and the triangle proper $A'B'C'$. If the length of the side of the triangle is regular $A'B'C'$ taken for 1, then, it can be shown that the area of the triangle $A_1'B_1'C_1'$, will be 7 times the area of the triangle ABC. A remarkable property of this problem is not only that it admits an optimal solution by the method of affine transformations, but also that it admits of generalization, and the generalization of this problem can be carried out in two directions: one of them is associated with the transition from one set to another more wide; containing the given set as a subset (from triangle to quadrangle, from quadrangle to pentagon, etc.), and the other direction is associated with the transition from parallelogram to parallelepiped by analogy. The ability of the future mathematics teacher to use the generalization method to compose new problems, tasks interrelated with this one, is one of the necessary conditions indicating his readiness to organize creative activity. This problem can be generalized to the case of quadrangles as follows: On the rays $AB,\ \ BC,\ \ CD,\ \ DA$ containing the corresponding sides of the parallelogram $ABCD$, points taken $D_1, A_1\ , B_1, C_1$ such that $BD_1 = AB,\ \ CA_1 = BC,\ \ DB_1 = CD,\ \ AC_1 = AD$. Find the ratio of the area of a parallelogram $ABCD$ to the area of the quadrangle $A_1B_1C_1D_1$.



First of all, when organizing the search for a solution to this problem, it is necessary to focus the attention of students on the possibility of determining the type of this quadrangle. Doesn't it, like this one, belong to the class of parallelograms? After simple reasoning, we can find that the quadrilateral $A_1B_1C_1D_1$ - parallelogram that consists of a parallelogram $ABCD$ and four triangles having the same area as this parallelogram. This means that the area of the parallelogram $A_1B_1C_1D_1$ five times the parallelogram area $ABCD$. This problem can be generalized to the case of an arbitrary quadrangle $ABCD$: on the rays $AB,\ \ BC,\ \ CD,\ \ DA$ points taken $D_1, A_1\ , B_1, C_1$ so that

$BD_1 = AB, \quad CA_1 = BC, \quad DB_1 = CD, \quad AC_1 = AD$. Find the area of the resulting quadrilateral $A_1B_1C_1D_1$ if the area of the given quadrangle $ABCD$ equals S.

It is known that a tetrahedron is a spatial analogue of a triangle, and a parallelepiped is a spatial analogue of a parallelogram. Using an analogy, a number of spatial problems can be compiled that are generalizations of previous planimetric problems, for example,

1.On the rays $AB, \quad BC, \quad CD, \quad DA$, containing the edges of the tetrahedron $ABCD$ points taken D_1, A_1, B_1, C_1 so, that $BD_1 = AB, \quad CA_1 = BC, \quad DB_1 = CD, \quad AC_1 = AD$. Find the ratio of the volume of a tetrahedron $ABCD$ to the volume of the tetrahedron $A_1B_1C_1D_1$.

2. On the rays

$$BA, B_1B, C_1C, CD, AA_1, C_1C, CD, AA_1, A_1B_1, D_1C_1, DD_1,$$

containing the corresponding edges of the parallelepiped $ABCDA_1B_1C_1D_1$, points taken $M, N, P, Q, M_1, N_1, P_1, Q_1$ such, that $AM = BA, BN = B_1B, CP = C_1C, DQ = CD, A_1M_1 = AA_1, B_1N_1 = A_1B_1, C_1P_1 = D_1C_1, D_1Q_1 = DD_1$. Prove that the volume of a polytope $MNPQM_1N_1P_1Q_1$ five times the volume of a parallelepiped $ABCDA_1B_1C_1D_1$.

One of the ways to find the optimal solution to these problems is based on the theorem on the geometric meaning of the mixed product of three non-coplanar vectors. According to this theorem, we find that V_{ABCD} = mod($\overrightarrow{C_1B_1}, \overrightarrow{C_1D_1}, \overrightarrow{C_1A_1}$)/6. Since vectors $\overrightarrow{C_1B_1}, \overrightarrow{C_1D_1}, \overrightarrow{C_1A_1}$ can be expanded in non-coplanar vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ in the following way:

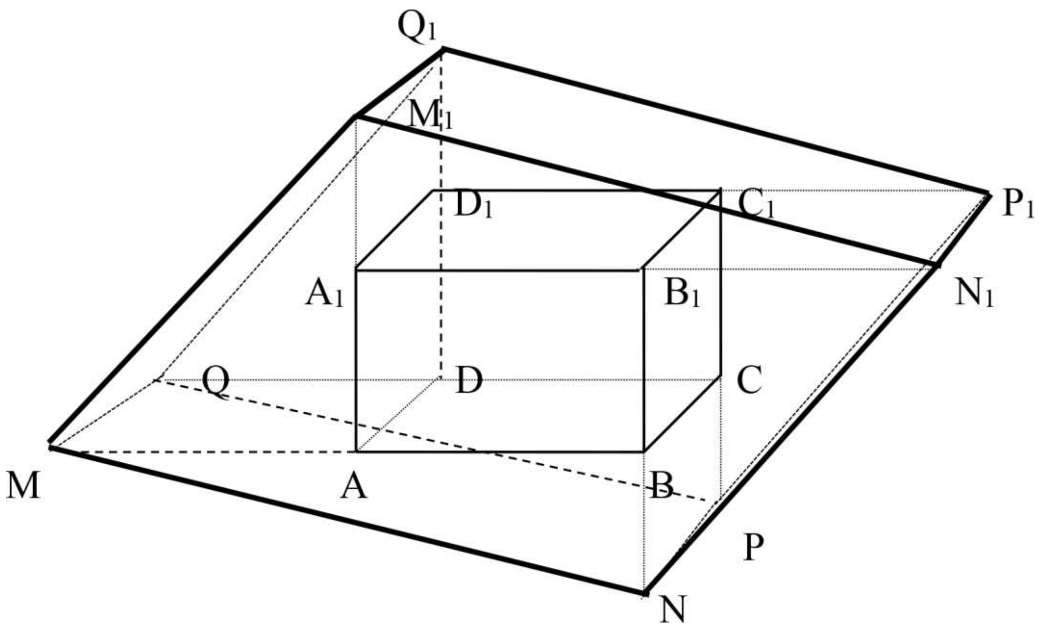
$$\overrightarrow{C_1B_1} = \overrightarrow{AC} - 3\overrightarrow{AB}, \quad \overrightarrow{C_1D_1} = -2\overrightarrow{AB} + 3\overrightarrow{AC} - \overrightarrow{AD},$$
$$\overrightarrow{C_1A_1} = -2\overrightarrow{AB} + \overrightarrow{AC} + 2\overrightarrow{AD},$$

then mixed product ($\overrightarrow{C_1B_1}, \overrightarrow{C_1D_1}, \overrightarrow{C_1A_1}$) of vectors $\overrightarrow{C_1B_1}, \overrightarrow{C_1D_1}, \overrightarrow{C_1A_1}$ will be expressed through the mixed product of vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ in the following way:

$$(\overrightarrow{C_1B_1}, \overrightarrow{C_1D_1}, \overrightarrow{C_1A_1}) = -15\,(\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}).$$

This means that the volume of the tetrahedron $A_1B_1C_1D_1$ 15 times the volume of this tetrahedron.

In order to prove that the ratio of the volume of the parallelepiped $ABCDA_1B_1C_1D_1$ to the volume of the parallelepiped $MNPQM_1N_1P_1Q_1$ equals 1 : 5, it is necessary to know the decomposition of vectors $\overrightarrow{MQ}, \overrightarrow{MN}, \overrightarrow{MM_1}$ by non-coplanar vectors $\overrightarrow{AD}, \overrightarrow{AB}, \overrightarrow{AA_1}$.



From the condition of the problem, considering the rules for adding vectors, we obtain that the vectors $\overrightarrow{MQ}, \overrightarrow{MN}, \overrightarrow{MM_1}$ related to vectors $\overrightarrow{AD}, \overrightarrow{AB}, \overrightarrow{AA_1}$ by the following ratios: $\overrightarrow{MQ} = \overrightarrow{AD}$, $\overrightarrow{MN} = 2\overrightarrow{AB} - \overrightarrow{AA_1}$, $\overrightarrow{MM_1} = \overrightarrow{AB} + 2\overrightarrow{AA_1}$. Therefore, taking into account the properties of the mixed product of vectors, we have $(\overrightarrow{MQ}, \overrightarrow{MN}, \overrightarrow{MM_1}) = -5(\overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{AA_1})$.. This means that the volume of the parallelepiped $MNPQM_1N_1P_1Q_1$ five times the volume of a parallelepiped $ABCDA_1B_1C_1D_1$.

When searching for an optimal solution to a geometric problem, it is important that future masters of mathematics education have developed the ability to represent a half-plane, the interior of a polygon, the interior of a circle and other geometric figures with appropriate algebraic models. In the formation of this skill among students, problems of the following type can be played: Inside an equilateral triangle, an arbitrary point is taken, from which perpendiculars are lowered to all its sides. Prove that the sum of the lengths of these perpendiculars is equal to the length of the height of the triangle. The search for a solution to this problem should begin with focusing the trainees' attention on the use of algebraic models of the corresponding geometric images. By the time this problem is studied, future masters of pedagogical education (profile "Mathematical education") should have the ability to link these geometric figures in a canonical way. The choice of a canonical coordinate system contributes to a significant reduction in computational activity. In this case, the canonical choice of the coordinate system is due to the fact that the center of any of the sides of the triangle can be taken as the origin of the coordinate system, for example, we take the middle O of the side AB. As the first coordinate vector, we take the unit vector co-directional with the vector $\overrightarrow{OB}$, and as the second coordinate vector we take the unit vector co-directional with the vector $\overrightarrow{OC}$. Without loss of generality, we can assume that the length of the side of an equilateral triangle is 2. Then, relative to a specially selected PDSK, points A, B, C will have the following coordinates: $A(-1;0),\quad B(0;\sqrt{3}),\quad C(1;0)$. Let M (x, y) be the interior point of the triangle. Using the coordinates of points A, B, C, you can draw up the equations of the lines containing the sides of this triangle. Then the interior of the triangle ABC will be determined by the system of inequalities:

$$\begin{cases} y \geq 0, \\ \sqrt{3}x - y + \sqrt{3} \geq 0, \\ \sqrt{3}x + y - \sqrt{3} \leq 0. \end{cases}$$

Using the formula for calculating the distance from a point to a line, you can show that

$$\rho(M, AC) = |y|, \quad \rho(M, BC) = \frac{|\sqrt{3}x + y - \sqrt{3}|}{2}, \quad \rho(M, AB) = \frac{|\sqrt{3}x - y + \sqrt{3}|}{2}$$

Considering the above inequalities, which are satisfied by the coordinates of the point M, we obtain that

$$\rho(M, AC) = y, \quad \rho(M, BC) = \frac{-\sqrt{3}x - y + \sqrt{3}}{2}, \quad \rho(M, AB) = \frac{\sqrt{3}x - y + \sqrt{3}}{2}$$

From where $\rho(M, AC) + \rho(M, BC) + \rho(M, AB) = \sqrt{3}$. Since the length of the height of the triangle is equal to the same, it means that the requirement of the problem is fulfilled. While solving such problems, future mathematics teachers master not only the methods of establishing a one-to-one correspondence between geometric figures and their algebraic models, but, first of all, master the methods of their application when searching for the optimal solution of a specific geometric problem of a school type.

The effectiveness of the impact of the educational environment on the health of primary students is determined by the systematic health activities. The process of formation a conscious attitude to one's own health requires the combination of information and motivation components with the students' practical activities, which will help them to acquire the necessary health-saving skills and habits (ROZLUTSKA *et al.,* 2020)

Conclusion

Thus, the methodological aspect of the task material presented in this article, aimed at preparing future masters of pedagogical education (profile "Mathematical Education") for creative activity, allows us to highlight the purposefulness of educational activity as a basic mechanism that ensures the effectiveness of the formation of competence, initiative and creativity. All these mechanisms act in unity and have a positive impact on the development of creative activity in the educational process. It was revealed that in the process of learning to solve geometric problems included in the developed system, students demonstrate higher indicators of the level of formation of creative activity.

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