POSSIBILITIES OF USING INFORMATION TECHNOLOGIES IN TEACHING MATHEMATICS (ON THE EXAMPLE OF A FUNCTIONAL-GRAPHIC METHOD FOR SOLVING EQUATIONS)

POSSIBILIDADES DE USAR TECNOLOGIAS DA INFORMAÇÃO NO ENSINO DA MATEMÁTICA (A EXEMPLO DE UM MÉTODO GRÁFICO FUNCIONAL PARA RESOLVER EQUAÇÕES)

POSIBILIDADES DE UTILIZAR TECNOLOGÍAS DE LA INFORMACIÓN EN LA ENSEÑANZA DE LAS MATEMÁTICAS (SOBRE EL EJEMPLO DE UN MÉTODO FUNCIONAL-GRÁFICO PARA RESOLVER ECUACIONES)

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ABSTRACT: The article is devoted to the methodology for the formation of a functional-graphic method in students using information technologies. The article describes the theoretical foundations of teaching the functional-graphic method for solving equations; methodological recommendations for the formation of students' actions that are adequate to this method with the use of software. To study the possibilities of using information technologies in teaching mathematical disciplines to students, to determine their effectiveness, the study was conducted at the Faculty of Physics and Mathematics of the Mordovian State Pedagogical University named after M. E. Evsevyev. The research process involved theoretical methods (the study and analysis of scientific and methodological, educational literature). Empirical methods were also used – monitoring the educational activities of students, the professional activities of teachers, surveys of the specified contingent were conducted.

KEYWORDS: Teaching mathematics. Information technology. Functional-graphic method. Equations.

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RESUMO: *O artigo é dedicado à metodologia para a formação de um método gráfico-funcional em alunos utilizando as tecnologias de informação. O artigo descreve os fundamentos teóricos do ensino do método gráfico-funcional para resolução de equações; recomendações metodológicas para a formação de ações adequadas dos alunos a este método com a utilização de software. Para estudar as possibilidades de utilização das tecnologias da informação no ensino de disciplinas matemáticas aos alunos, para determinar a sua eficácia, o estudo foi realizado na Faculdade de Física e Matemática da Universidade Pedagógica do Estado de Mordóvia em homenagem a M. E. Evsevyev. O processo de pesquisa envolveu métodos teóricos (estudo e análise da literatura científica e metodológica educacional). Também foram utilizados métodos empíricos - monitoramento das atividades educacionais dos alunos, das atividades profissionais dos professores, pesquisas com o contingente especificado.*

PALAVRAS-CHAVE: *Ensino de matemática. Tecnologia da informação. Método gráfico-funcional. Equações.*

RESUMEN: *El artículo está dedicado a la metodología para la formación de un método gráfico funcional en estudiantes utilizando tecnologías de la información. El artículo describe los fundamentos teóricos de la enseñanza del método gráfico funcional para la resolución de ecuaciones; recomendaciones metodológicas para la formación de acciones de los estudiantes que se adecuen a este método con el uso de software. Para estudiar las posibilidades del uso de las tecnologías de la información en la enseñanza de disciplinas matemáticas a los estudiantes, para determinar su efectividad, el estudio se llevó a cabo en la Facultad de Física y Matemáticas de la Universidad Pedagógica Estatal de Mordovia que lleva el nombre de M. E. Evsevyev. El proceso de investigación involucró métodos teóricos (el estudio y análisis de literatura científica y metodológica, educativa). También se utilizaron métodos empíricos: se monitorearon las actividades educativas de los estudiantes, las actividades profesionales de los maestros, se realizaron encuestas del contingente especificado.*

PALABRAS CLAVE: *Enseñanza de matemáticas. Tecnología de la información. Método gráfico funcional. Ecuaciones.*

Introduction

The effectiveness of the use of information technologies in the educational process of schools and universities, including in teaching mathematical disciplines, has been studied by several authors (KOCHETOVA; SARVANOVA; PORVATKIN, 2021; DERBEDENEVA *et al.*, 2018; SIMSEK; BALABAN, 2010; SHUKSHINA; HULL; RYZHOV, 2020).

The works of many mathematicians and methodologists are devoted to issues related to the formation of students' ability to apply the functional-graphic method for solving equations. The unifying feature of these works is the consideration of a limited range of relevant methodological aspects.

V. I. Golubev (2007) noted the main regularities that govern the processes of solving equations using the functional-graphic method. They are associated with the specific operations of the student using this method and are expressed in an activity form.

Let us briefly outline the composition of the main operations that should be formed in the student of the application of the functional-graphic method for solving equations:

- search for signs that allow attributing an equation to an already known class of equations;

- checking the existing record of the equation for compliance with the standard form and bringing it to such (if required);

- writing the equation in the form $q(x) = h(x)$ in several versions;

- drawing up functions of the form $y_1 = q(x)$; $y_2 = h(x)$ based on the results of the previous operation;

- plotting two well-known functions from the same group;

- studying the graph for the presence of roots and their number, getting an answer.

Obviously, this list of actions is incomplete and can be changed depending on the existing equation, but these structural elements must be presented when solving.

V. I. Golubev presents the correspondence between the above operations and the cognitive universal educational actions formed during their implementation.

S. V. Akmanova (2020) considers the formation of actions corresponding to the functional-graphic method of solving equations as an effective means of preparing for the final exams in mathematics. In her works, both examples of specific tasks and the structure of signs are given by which it is possible to evaluate tasks to subsequently compose a didactically and methodologically correct sequence for teaching students to solve problems with a parameter.

The originality of the view of E.V. Bazhenova (2018) on the problem of the formation of the method under consideration in students is reflected in the recommendations for the selection of tasks. Among the whole set of the latter, there must necessarily be those that do not require the construction of graphs of functions. In this case, students should at least at a visual level own ideas about the properties of the functions selected during the solution. After solving the problem in one way, they should be asked to independently substantiate the irrationality or inefficiency of the other, and in some cases, on the contrary, its advantages.

A. V. Imanova (2018) emphasizes the importance of developing students' ability to independently compose tasks for the application of the functional-graphic method to solving equations. In her work, a methodological scheme is presented that allows the teacher to organize the above process. Its main components are:

- updating knowledge about the features of the application of the functional-graphic method to solving an equation of this type;

- solution of the proposed equation;

- analysis of the drawing proposed by the teacher with graphs of functions illustrating the solution of an equation of this type unknown to students;

- self-compilation by students of the equation;

- analysis of the set of techniques underlying the solution of the functional-graphic method of an equation of this type with the allocation of the central (reference) technique.

The groups of techniques, which are the components of this method, are described. Each of them corresponds to the class of tasks selected by the author. The advantage of the proposed scheme of interaction between the teacher and students is the possibility of a qualitative check of the educational result based on the universal criteria specified by the author.

A number of methodological aspects of teaching students the functional-graphic method for solving equations using software tools were developed by T. A. Almazova (2019), E. E. Detterer (2017).

The first author (ALMAZOVA *et al.*, 2019) highlighted a sequence of basic actions that are adequate to the functional-graphic method for solving equations and formed using the GeoGebra mathematical software tool. On its basis, a group of exercises was compiled, corresponding to the type of «practical task - questions to the task». The advantages of this approach are obvious. These include: a visual basis for learning; a high degree of independence of students' actions. The disadvantage of this approach is the lack of clarity in the principle of combining machineless and machine-free variants of tasks and their alternation. In the specific examples of groups presented by the author, the named tasks are arranged in a chosen way without justification.

E. E. Detterer (2017) presents examples of using the GeoGebra program in teaching solving problems with parameters at the initial stage. The author proposed a scheme for the step-by-step solution of this type of equations by the functional-graphic method. This organization of the solution to the equation allows developing the ability of students to visualize how the coordinate plane with the solution should look approximately like, and this, in turn, is also necessary to concretize the areas of variation of the parameter that need to be investigated.

Materials and methods

To study the possibilities of using information technologies in teaching mathematical disciplines to students, to determine their effectiveness, the study was conducted at the Faculty of Physics and Mathematics of the Mordovian State Pedagogical University named after M. E. Evsevyev. The research process involved theoretical methods (the study and analysis of scientific and methodological, educational literature). Empirical methods were also used – monitoring the educational activities of students, the professional activities of teachers, surveys of the specified contingent were conducted. The study was based on the data obtained by the authors as a result of testing the developed methods and means of teaching students of the Faculty of Physics and Mathematics at lectures and practical classes during distance learning.

The materials of the article can be used by teachers of methodological disciplines of pedagogical universities, have practical significance, will allow to realize the professional orientation of training.

Results and discussions

We describe the teaching methodology for the functional-graphic method of solving equations using software.

Here are the actions, the formation of which forms the basis of teaching the functional-graphic method of solving equations as part of the work on developing students' ability to build and analyze the resulting graphs of functions:

- graphing functions; assessment of the possibility of the presence of the roots of the equation on a given interval;

- identification of points of intersection of graphs of functions;

- revealing the monotony of the function;

- assessment of the mutual relation of the sets of values of two functions;

- investigation of the behavior of the function for various values of the parameters included in the formula that defines it;

- identification of the values of the parameters included in the original equation, at which the functions compiled on the basis of this equation have common points;

- establishment of dependencies between graphical images of functions, allowing to reduce the solution of this equation to the consideration of a particular case of configuration for a given choice of parameters.

Next, we present the composition of operations, the formation of which with the use of software is the main part of the work on teaching students the functional-graphic method for solving equations:

1) compilation of groups of formulas that define functions based on this equation;

2) assessment of graphic images that can potentially be obtained when plotting graphs for each group in order to select the most accurate image suitable for the condition of obtaining;

3) analysis of the selected group: highlighting the variable parameters included in each formula;

4) assessment of the permissible limits of variation of the parameters;

5) declaration of variable parameters in the program;

6) setting the initial and final values of the variable parameters in accordance with the selected permissible variation limits;

7) setting functions in the program (by writing a formula in the input line) and sending a command to build a graph (if the latter is necessary due to the peculiarities of the software);

8) assessment of the quality of the resulting image - if it is impossible to set the intersection points, it is necessary to adjust the display scale or parameter values;

9) setting the value of the abscissa of the intersection points of the graphs - by hovering the mouse cursor or using the appropriate tool in the program (if available);

10) conducting a study of image changes by changing the parameter values (if necessary, depending on the purpose of the study).

The teacher's activities aimed at teaching the method under consideration using information technology should be organized within the framework of the formation of the above actions. It is quite possible that some elements of the last list will be introduced by students into the algorithm for solving equations by the functional-graphical method using the selected program. In general, these actions are mandatory (pivotal), but not the only ones.

Let us describe the components of the functional-graphic method for solving equations by presenting them in the form of the results of solving the tasks presented below.

An overview of an example of the application of the functional-graphical method for solving equations

Students should be invited to familiarize themselves with the solutions of several equations by the method under consideration, and then highlight the sequence of actions necessary to solve. It is necessary to invite students to independently divide the entire solution process into stages, enter them into a special table with a justification for the purpose of the actions being performed.

Let us present the formulations of tasks adequate to this stage.

The exercise 1. Get acquainted with the solution of the equation by the functional-graphical method.

The exercise 2. Carefully study the presented process of solving the equation by the functional-graphic method. Highlight the sequence of steps in the solution.

The exercise 3. Review the steps for each of the steps you have highlighted.

The exercise 4. Answer the questions.

1. What is the outcome of each highlighted stage?

2. What is the purpose of the actions at each stage?

The exercise 5. Using the software tool, complete the constructions shown in the example.

Drawing up an algorithm for solving equations by the functional-graphic method

At this stage, students should, in addition to the content of the previous stage, analyze 2-3 examples of solving equations by the method under study. At the same time, the conclusions obtained should be correlated with the results of completing exercises 2-4, if possible, pronouncing the reasons for the differences in actions, if any. While working with examples, the teacher should ask students questions that would lead them to the idea of the presence of commonality in the actions of solving equations, which should be selected in such a way that working with them would involve the use of different techniques that are adequate to the method under consideration.

The exercise 6. Consider examples of solving equations by the functional-graphical method. Compare the compositions of actions that make up the solution. Identify the similarities and differences, explain the existence of the latter.

The exercise 7. Make an algorithm for solving equations by the functional-graphical method.

The exercise 8. Write a description of the sequence of actions when working with the software tool when solving an equation according to the algorithm you compiled in the previous task.

Applying the algorithm to solving equations

Before applying the compiled algorithm to solving equations by a functional-graphic method using a software tool, additional work should be carried out with the students to form the actions of preparing the equation, bringing it to an acceptable form for applying the method. It is also advisable to organize this activity in stages, which we will characterize further.

Drawing up formulas that define functions based on a given equation

When considering the possibility of solving an equation of this type by the functional-graphic method, one should first assess the objective ability of students to compose on its basis such functions that would already be familiar to them. It is also obvious that several pairs of functions can be composed from this equation. Among them should be those that students have already studied at least at a visual level without a detailed study of their properties.

Let us present the approximate structures of tasks that can be offered to students at this stage.

The exercise 9. Consider the equation. Write down the general view of this equation.

The exercise 10. Grouping the terms in the parts of this equation, separated by the «=» sign, write down all possible sets of the form $\begin{cases} y = f_1(x) \\ y = f_2(x) \end{cases}$.

The exercise 11. Among the sets that you recorded during the previous task, write down those that contain only the functions you have studied. Determine the type of functions in each case.

The exercise 12. Select one of the sets you recorded. Determine which lines represent the graphics for each function. Sign the line name next to the corresponding dialing function formula.

Selection of rational options for setting the initial parameters for the software tool

We will present examples of tasks that will allow the student to review the options for organizing the solution, quickly assess their labor intensity and choose the most suitable one.

The exercise 13. Think over the options for entering the initial data into the program for plotting the graphs of functions. Write down each option.

The exercise 14. Estimate the number of actions corresponding to each case recorded during the previous task. Choose the method with the least amount of action.

Solving an equation using a software tool

If the volume of operations is such that it is difficult for students to adhere to the intended sequence of actions for working with the program, they can refer to the algorithmic prescription they compiled when completing task 8.

The exercise 15. Solve the original equation using the program. Indicate its roots as an answer.

The exercise 16. Solve the equations using the functional-graphical method.

Integration of knowledge from this and previous topics

The work of students at this stage can be organized in different ways, depending on whether the programs for working with function graphs were used in the study of the previous topics or not.

In the first case, students can be offered tasks for highlighting actions with the program that were used when performing operations within the framework of previous lessons, and also ask clarifying questions, the purpose of which is to identify commonality in solving tasks. In addition, the teacher can prepare an exercise in which students will have to draw up a graphical flowchart connecting the actions of this topic and the previous ones.

In the second case, students should be offered tasks to determine the relationship of the actions that underlie this method with the previously studied content.

After the students have formed the ability to apply actions that are adequate to the functional-graphic method of solving equations, they can proceed to the formation of their groups. For this purpose, tasks should be selected, which, on the one hand, would involve repetition of previously studied content on the topic, and on the other, mastering new ways of working with the equation and functions derived from it.

Conclusion

The analysis of the descriptions of the process of forming the elements of the functional-graphic method for solving equations using information technologies in the scientific and methodological literature made it possible to identify approaches to the organization of teaching the functional-graphic method for solving equations presented in the scientific literature. The first is based on the use of a group of tasks, compiled based on the selected actions, adequate to the method, more, in a standard situation. In this case, the software acts as a means of reducing the time spent on the solution. In this case, the tasks are accompanied by a set of questions, according to which the student can check whether his work is moving in the right direction. The second approach is based on the use of practice-oriented tasks, the solution of which involves the preparation of equations containing a parameter. Thus, students will need not only to compose function formulas from this equation, but also to explore each of them using transformations using the capabilities of the selected software tool, since some transformations cannot be performed without involving the capabilities of computer technology.

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